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DEPLOYMENT OF THE ROTATING TWO BODIES TETHER IN AN ORBIT

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Предметом дослідження є невелика орбітальна зв'язка, що складається з двох тіл, які мають бути розгорнуті з космічного апарата таким чином, щоб після завершення розгортання вона оберталася з постійною кутовою швидкістю та мала задану довжину. Маса кінцевих тіл вважається однаковою, а трос, який з'єднує ці тіла, розглядається як безмасовий. Метою дослідження є розробка програми управління довжиною троса з урахуванням впливу на його динаміку гравітаційного моменту центрального ньютонівського силового поля. Для розгортання використовується попереднє обертання зв'язки навколо бінормалі орбіти. Дослідження складається з двох етапів. На першому етапі розробляється закон управління довжиною троса для досягнення бажаного розгортання. На цьому етапі використовуються рівняння руху зв'язки, записані в сферичних полярних координатах для спеціального випадку руху зв'язки в площині орбіти. На другому етапі проводиться числове моделювання динаміки розгортання троса під дією розробленого програмного закону управління, зосереджуючи увагу на зміні довжини зв'язки. Математична модель зв'язки враховує повний набір рівнянь руху зв'язки зі змінною довжиною в сферичних полярних координатах, що описують її тривимірний рух. Сила натягу тросу в цих рівняннях виражена через запрограмований закон зміни довжини та його перші дві часові похідні. Новизна дослідження полягає в формулюванні програмного закону управління, де трос розглядається як малоприводна (underactuated) механічна система. У дослідженні використовувалися методи аналітичної механіки, числові методи та методики, розроблені самими авторами. Отримані результати дозволяють встановити діапазони значень параметрів розгортання, які забезпечують виконання такого типу операцій. Практична значимість результатів полягає у можливості розгортання невеликих зв'язок на орбіті та приведення їх до стану рівномірного обертання із заданою довжиною за допомогою контролю довжини.

Ключові слова: *космічний трос, розгортання, обертання, динаміка, управління.*

The focus of the study is a compact orbital tether consisting of two bodies that must be deployed from a spacecraft so that after deployment it rotates at a constant angular speed and attains a prescribed length. The end masses are taken as equal, and the connecting cable is assumed massless. The aim of the research is to develop a feedforward control for the tether length, taking into account the gravitational torque exerted by the central Newtonian field of forces. Initial rotation of the tether about the orbit's binormal is employed for deployment. The research consists of two stages. The development of the control law is based on the tether length in the first stage, which provides the performance of the desired deployment. The tether motion equations, expressed in spherical polar coordinates for a particular case of tether motion in the orbital plane, are used at this stage. In the second investigative phase, a numerical simulation of the deployment dynamics of a tether under the effect of the developed feedforward control law is performed using the tether length. As a mathematical model of the tether, the full set of motion equations for a variable-length tether in spherical polar coordinates is employed, describing the spatial motion of bodies. The tensile force of the cable entering these equations is described by the control law governing length variation and its first two time derivatives. The novelty of this study lies in devising a feedforward control law that treats the tether as an underactuated mechanical system. Analytical mechanics, numerical techniques, and authors-developed methods were employed in this research. The results enable determination of the permissible ranges of deployment parameters, facilitating this type of deployment. Practically, the findings allow small tethers to be deployed in orbit and brought to uniform rotation at a specified length via tether length control.

Keywords: *space tether, deployment, rotation, dynamics, control.*

Introduction. The use of tethers represents one of the promising directions in the current development of space technologies. Cable systems are regarded as attractive for solving numerous tasks in space [1 – 11]. Particular interest is drawn to centrifugal systems, whose deployment and shape maintenance are achieved through the centrifugal forces generated when a structure rotates about its center of mass. In recent years many projects have employed centrifugal forces to preserve the shape of space antennas. Since there are no drag forces in space, maintaining a constant rotational speed of bodies is relatively straightforward. Although theoretical studies examine the contours of extremely large tethers, researchers continue to focus mainly on small tethers, typically supporting systems ranging from a few meters to several kilometers in length and having masses from a few kilograms up to a few hundred kilograms [4].

Such systems are of practical relevance because they are suitable both for solving independent scientific tasks and for implementing full-scale experiments and checks of a number of hypotheses concerning tether deployment dynamics and methods of creating their steady motions. For instance, a compact rotating tether can serve as a length standard for calibrating onboard optical and radar servo-mechanisms, as a built-in sensor of the planetary force field, and so on.

One possible implementation of a small-size space tether (a few dozen meters long) is a system of two bodies linked by an elastic cable. Typically, the masses of the end bodies determine the total mass of the tether (Fig. 1); the mass of the cable connecting the ends usually accounts for no more than a percent of the total tether mass. As a model, one may choose two equal point masses joined by a flexible, massless cable. The dimensions of the end bodies can be neglected when studying motions in which the cable cannot be wound onto the end bodies.

The simplest and most economical approach, both in construction and in mathematical description of deployment in orbit, is to rely on centrifugal forces. These forces can arise from an initial rotation of the system in the plane that passes through the centers of mass of its end bodies. Two actuators (for example, springs) must be installed on the spacecraft for this purpose; on command from Earth or from the spacecraft's onboard computer, they push the system away. As it separates from the spacecraft, it begins to rotate in the preset plane relative to the spacecraft–normally the orbital plane.

The initial angular velocity must be high enough to ensure, on the one hand, deployment of the system by centrifugal forces and, on the other hand that the resulting angular velocity and tether length correspond to the intended operational duty.

A similar problem was examined in the scientific literature [1] without considering the gravitational field. Deployment of a three-body tether was performed in a centrifugal-force field with tension control. That control was proposed to be realized by generating viscous and Coulomb friction forces that resist pulling the cable from the middle body, depending on deployment length and speed. Here we consider the deployment problem under orbital flight conditions for a two-body tether with control over the length of the connecting cable.

We introduce an orbital reference frame $\mathrm{C}x^{or}y^{or}z^{or}$ with its origin at the tether's center of mass C, where the $\mathrm{C}x^{or}$ axis points along the local vertical, $\mathrm{C}y^{or}$ aligns with the orbital velocity vector, and $\mathrm{C}z^{or}$ is normal to the orbital plane (Fig. 1). This frame rotates about the $\mathrm{C}z^{or}$ axis at a constant angular velocity ω^{or}.

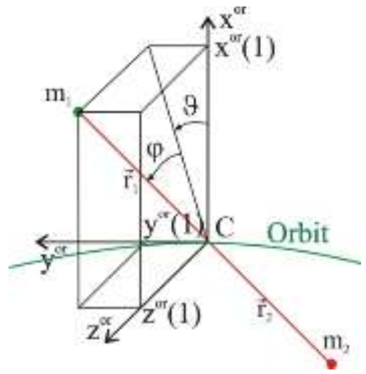

Fig. 1

The mathematical description of the process of ejecting a system as a rigid body from the spacecraft is rather simple, and we will not dwell on it. If we consider the motion of the system since the instant of its separation from the actuator, its mechanical model represents a free system of two bodies tied by a cable, undergoing relative motion under the influence of centrifugal and gravitational forces. We will assume that the ejection system will not cause motion of the bodies about their centers of mass at the moment of separation from the actuators. In that case the terminal bodies can be treated as point masses. Let us further assume that this condition is satisfied. The cable linking the bodies will be regarded as elastic because at the initial stage of rotation the tension in the cable can be quite large, and therefore it is essential to account for its elastic deformations when controlling the tether length. We will assume deployment of the tether from one of the bodies so that the masses of the bodies are equal at the end of deployment.

Since the dimensions of the considered tether are substantially smaller than the size of the orbit, the problem statement can be "limited" [3]; the equations of motion of the tether's center of mass are decoupled from the equations concerning the spacecraft's center of mass in this case. Consequently, we will focus solely on studying the system's attitude. When the tether is ejected from the spacecraft by impulses of varying magnitude, its center of mass acquires an initial translational velocity. As a result, the tether's center of mass moves in inertial space relative to the spacecraft.

At the same time, the motion of the deploying system occurs in a region of space that must be free of obstacles. Therefore, when simulating the tether-deployment dynamics, it is helpful to define the limits of such a region. This issue is trivial and of little scientific interest. Hence, we will concentrate further only on the rotational motion of the tether in the orbital frame of reference.

Assume the deployment of the tether takes place in the orbital plane. Let the distance between the centers of mass of the tether's end bodies be its length. Denote the tether length at the initial instant by L_0 and its final length by L_F. When the end masses are equal, it is sufficient to examine the motion of a single end body. In planar motion the state of the point mass can be described by its distance r from the tether's center of mass and the pitch angle ϑ, measured in the orbital plane from the local vertical.

Typically a single control input associated with varying the tether length or its tension is used in models of real tethers with a massless cable whose motion lies in the plane of a circular orbit, yet the system possesses two degrees of freedom. The second degree of freedom is associated with the pitch rotational motion of the tether. In this formulation the tether can be regarded as an under-actuated mechanical system, where the number of degrees of freedom exceeds the number of control inputs. We cannot directly influence the pitch angle, so it is necessary to devise an appropriate control law for the tether length that produces the desired pitch-angle dynamics.

Our goal is to find a control law $L(t)$ that achieves deployment of the tether in the orbital plane to the desired length and a prescribed rotational condition while avoiding longitudinal oscillations.

To derive the needed control laws, one may employ the angular-momentum conservation theorem for the tether. A simpler approach is to employ the motion equation for the pitch angle of an inextensible tether, whose length can vary via unwinding. Owing to the monotonic nature of the deployment process in this model, it is straightforward to introduce corrections for the tether length due to elastic deformation. Following [4] (pp. 65), the equation for the planar motion of a tether in a circular orbit can be written as follows:

$$\ddot{\vartheta} + 2(\dot{\vartheta} + \omega^{or})\dot{r} / r + 3(\omega^{or})^2 \sin\vartheta \cos\vartheta = 0 \tag{1}$$

here ω^{or} is the orbital angular velocity; $r(t) = L(t)/2$ is half of the tether length; ϑ is the pitch angle.

Although we assume that the tether during deployment will be displaced relative to the spacecraft, the speed of its motion is negligibly small compared with the spacecraft's velocity in the inertial frame. Consequently, the angular velocity of the tether's motion in orbit can be taken as identical to that of the spacecraft.

To develop the feedforward controlling tether deployment to a specified length while maintaining uniform rotation at the desired angular velocity, we will employ the method described in [0, 11].

After elementary manipulations of equation (1), one can obtain a first-order ordinary differential equation together with the appropriate initial condition.

$$\dot{L} = -L\frac{3(\omega^{or})^2 \sin 2\vartheta + 2\ddot{\vartheta}}{4\left(\omega^{or} + \dot{\vartheta}\right)}, \; L(0) = L_0 . \tag{2}$$

Its solution can be written down as the following:

$$L(t) = L_0 \exp\left[-\int_0^t \left(\frac{3(\omega^{or})^2 \sin(2\vartheta(\tau)) + 2\ddot{\vartheta}(\tau)}{4\left(\omega^{or} + \dot{\vartheta}(\tau)\right)}\right) d\tau\right]. \tag{3}$$

It follows from equation (3) that $L(t)$ is a function of the pitch angle $\vartheta(t)$, its first two time derivatives, and the duration of the maneuver $t_F = T_F$. It is clear that the variation law for the pitch angle and its time derivatives must satisfy certain conditions for the deployment mode under consideration, both at the start of deployment and at its conclusion. We now examine how to select such boundary conditions on physical grounds.

We choose a variation law for the angle $\vartheta(t)$ that yields a smooth deployment of the tether to a prescribed constant length. Numerous possible laws $L(t)$ for the tether-length change can be generated for different values of the maneuver duration

T_F . From this family of solutions, one can be selected that achieves the desired final tether length, avoids loss of tension during deployment, and satisfies the additional requirements that will be stated below.

Next, we consider the constraints that must be imposed on the pitch-angle variation law $\vartheta(t)$ in order to define the time-dependent tether-length law $L(t)$ that permits solving the problem via the Cauchy-problem solution (2).

As mechanisms for generating the initial rotation of the tether, two spring pushers acting in opposite directions with different impulses may be employed.

For the initial condition of the tether deployment, we assume it is aligned with the local vertical, possessing a prescribed angular velocity (both magnitude and direction) and an initial translational velocity of the tether's mass center in the orbital reference frame. In this case the following initial conditions for the pitch angle and its angular velocity can be written:

$$\vartheta(0) = 0, \quad \dot{\vartheta}(0) = \dot{\vartheta}_0, \tag{4}$$

and an initial condition for the relative velocity of the tether mass center in the orbital frame of reference:

$$v_C = v_0 . \tag{5}$$

As in an initial instant the contacts of bodies of a tether with the device of creation of its initial motion stop, at this instant both the angular and linear accelerations turn into zero, i.e.,

$$\ddot{\vartheta}(0) = 0 \ , \tag{6}$$

$$\dot{v}_C = 0 . \tag{7}$$

The following conditions have to be satisfied at the initial instant of deployment and at the time of its termination;

$$L(0) = L_0 \, , L(T_F) = L_F \, . \tag{8}$$

The first of these conditions can be regarded as satisfied; the second condition can be used further to define the a priori unknown time at which the maneuver ends. This value will enable the selection of the appropriate solution of the problem from the set of solutions obtained for various values.

Two additional conditions follow from the requirement of constancy of the tether length at the initial and final instants:

$$\dot{L}(0) = 0 \, , \dot{L}(T_F) = 0 \, . \tag{9}$$

The emergence of jumps in the tension of the cable is unacceptable at the initial and final instants of the maneuver and throughout the entire maneuver, because they can lead to a loss of tension that would render the adopted mechanical model inadequate. The absence of tension jumps at the initial and final instants is ensured when the following conditions are satisfied:

$$\dot{L}(0) = 0 \, , \dot{L}(T_F) = 0 \, . \tag{9}$$

To obtain the restriction on the law governing the change of the pitch angle from these conditions, we differentiate equation (2) with respect to time:

$$\ddot{L} = 1/[4\left(\omega^{or} + \dot{\vartheta}\right)^2]\{-2\dot{L}\left(\omega^{or} + \dot{\vartheta}\right)[3(\omega^{or})^2 \sin 2\vartheta/2 + \ddot{\vartheta}] + L\ddot{\vartheta}[3(\omega^{or})^2 \sin 2\vartheta + 2\ddot{\vartheta}] - 2L\left(\omega^{or} + \dot{\vartheta}\right)[3(\omega^{or})^2 \dot{\vartheta}\cos 2\vartheta + \dddot{\vartheta}]\}. \quad (11)$$

At the initial instant, using conditions (4, 6, 8 – 10), expression (11) becomes

$$\ddot{L}(0) = -L_0[6(\omega^{or})^2 \vartheta_0 + 2\dddot{\vartheta}(0)]/4/\left(\omega^{or} + \dot{\vartheta}\right) . \quad (12)$$

Considering condition (10) yields an additional requirement that must be satisfied when the pitch angle varies with time:

$$\dddot{\vartheta}(0) = -3(\omega^{or})^2 \dot{\vartheta}_0 . \quad (13)$$

Next, we examine further constraints that must be imposed on the law $\vartheta(t)$ at the final instant $t = T_F$ of tether deployment in the designated state. At the final state, when the deployment process ends, it is convenient to assume that the tether has rotated an integer number of turns relative to its initial position:

$$\vartheta(T_F) = 2\pi n_{rot} . \quad (14)$$

Hence, the tether at $t = T_F$ will be aligned with the local vertical. Since the gravitational torque vanishes at this moment and no other external forces act on the tether in the present formulation, we can introduce the additional condition

$$\ddot{\vartheta}(T_F) = 0 . \quad (15)$$

Rather than constraining the final tether length, we prescribe its final angular velocity.

$$\dot{\vartheta}(T_F) = \dot{\vartheta}_F . \quad (16)$$

The final tether length can be selected, if required, by adjusting the value $\dot{\vartheta}_F$. Finally, using condition (15) and by analogy with (13), we obtain another condition

$$\dddot{\vartheta}(F) = -3(\omega^{or})^2 \dot{\vartheta}_F . \quad (17)$$

Consequently, the law $\vartheta(t)$ must satisfy eight conditions (4), (6), and (13 – 17). The required law $\vartheta(t)$ can be expressed as any suitable series of functions, with its coefficients determined from the eight constraints. Here we choose a seventh-order power series for the law $\vartheta(t)$.

$$\vartheta(t) = \sum_{i=0}^{7} c_i \left(\frac{t}{T_F}\right)^i . \quad (18)$$

At satisfaction of eight specified boundary conditions for the law $\vartheta(t)$ the expression for coefficients of the power series (18) take the following form:

$$c_0 = 0,$$
$$c_1 = \vartheta_0,$$
$$c_2 = 0,$$
$$c_3 = -(\omega^{or})^2 \dot{\vartheta}_0 / 2,$$
$$c_4 = -[-140 n_{rot}\pi + 40\dot{\vartheta}_0 T_F + 30\dot{\vartheta}_F T_F - 4(\omega^{or})^2 \dot{\vartheta}_0 T_F^3 - (\omega^{or})^2 \dot{\vartheta}_F T_F^3] / 2 / T_F^4.$$
$$c_5 = -3[112 n_{rot}\pi - 30\dot{\vartheta}_0 T_F - 26\dot{\vartheta}_F T_F + 2(\omega^{or})^2 \dot{\vartheta}_0 T_F^3 + (\omega^{or})^2 \dot{\vartheta}_F T_F^3] / 2 / T_F^5.$$
$$c_6 = -[-280 n_{rot}\pi + 72\dot{\vartheta}_0 T_F + 68\dot{\vartheta}_F T_F - 4(\omega^{or})^2 \dot{\vartheta}_0 T_F^3 - 3(\omega^{or})^2 \dot{\vartheta}_F T_F^3] / 2 / T_F^6. \qquad (19)$$
$$c_7 = -[80 n_{rot}\pi - 20\dot{\vartheta}_0 T_F - 20\dot{\vartheta}_F T_F + (\omega^{or})^2 \dot{\vartheta}_0 T_F^3 + (\omega^{or})^2 \dot{\vartheta}_F T_F^3] / 2 / T_F^7.$$

Now, having set parameters of the problem and having substituted their values in the coefficients (19), we will obtain the law of change of pitch angle in time corresponding to the required mode of the motion of the tether at its deployment from an initial state in the state of rotation with the set angular velocity. The law of change of the tether length in time, which implements the required mode of the tether deployment, can be obtained from the solution of the Cauchy problem (3), having substituted laws of change of the pitch angle and its time derivatives there. Conditions (5) and (7) define only the translatory motion of the tether mass center in the orbital frame of reference and have no influence on its attitude.

Numerical research. Let us consider further implementation of the stated method of creation of the feedforward control of the space tether deployment in the condition of uniform rotation in the orbit plane. Let us choose the following basic values of the tether parameters:

The masses of the end bodies are 10 kg.

The duration of the deployment process is 50 s.

The initial length of the tether L_0 is 4 m.

The initial angular velocity of the tether ϑ_0 = 15 rad / s.

The final length of the tether L_F is 15 m.

The longitudinal rigidity of the cable EF is 26104 N.

The velocity of leaving of the center of mass of the tether v_0 is 0.2 m/s,

The basic value of the number of turns n_{rot} is 70.

The radius of the orbit of the tether is 7000 km.

Let us show further how to find the law, which solves the problem.

Apparently, from expressions (2) and (19), the required control law is defined by values of such parameters as the number of turns of the tether during deployment n_{rot}, the initial value of the pitch angle of the tether ϑ_0, the initial value of the angular velocity $\dot{\vartheta}_0$ of rotation of the tether in the orbital frame of reference, the final value of the angular velocity $\dot{\vartheta}_F$ of the tether rotation in the orbital frame of reference, and the deployment process duration T_F. These parameters define the law of change in time of the tether pitch angle $\vartheta(t)$ according to expression (18). The initial value of the semi-length of the tether r_o is included directly in the control law (3) of program change of the tether length.

In the beginning we will estimate the influence of the parameter n_{rot} on the nature of the required law of feedforward control, the tether length, and the behavior of other characteristic values. In Fig. 2 the change of shape of the control law $L(t)$ depending on values of the parameter n_{rot} at basic values of other parameters is shown.

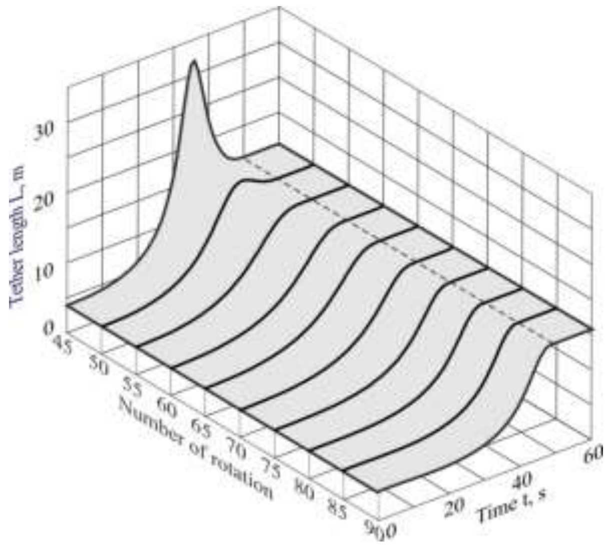

Fig. 2

Here the number of whole revolutions of the tether during the deployment n_{rot} changes from 45 to 90. In all cases the length of the tether deployment is equal; nevertheless, the nature of its change considerably depends on the accepted number of turns. In the range from 45 to 60 turns at approach to the end of deployment, the tether length becomes more than the final length. For the realization of such a mode in the device of deployment, it will be required to carry out not only the controlled deployment of the cable but also its controlled retrieval. Therefore, for practical implementation, the range of values $n_{rot} > 60$ is represented the most appropriate. The straight dashed line on it and other figures corresponds to the time of the deployment completion $t = 50$ s.

Fig. 3 shows the program law of change in time of the tether length in the case of rigid cable.

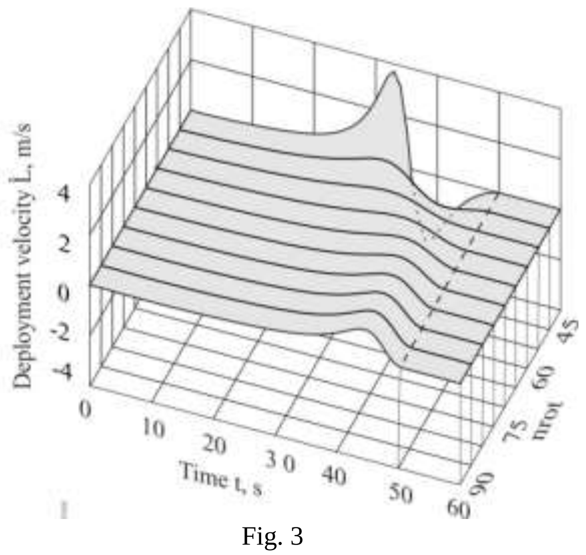

Fig. 3

In this figure the parts of curves where the velocity of exit of the cable has to be negative are good visible $n_{rot} \in [45, 60)$. Most strongly this effect is expressed at $n_{rot} = 45$. At the same time, such parts of the curves corresponding to the condition $n_{rot} > 80$ are expressed less considerably.

The behavior of the pitch angle of the tether in time depending on value n_{rot} is shown in Fig. 4.

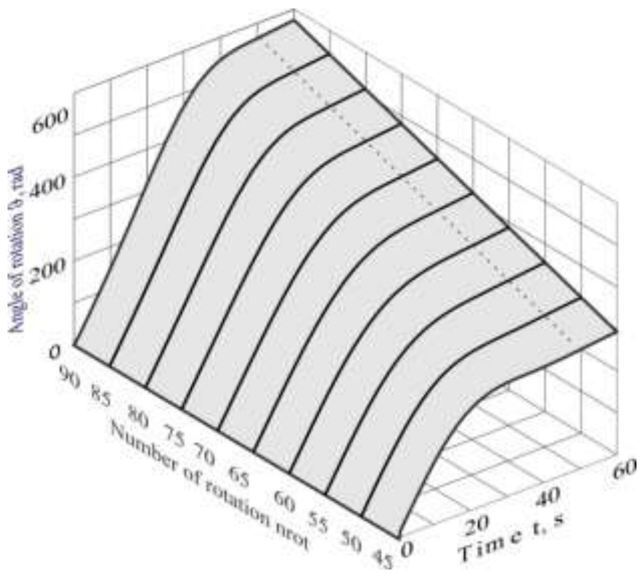

Fig. 4

Here the direct dependence between the number of turns and the change of the angle of rotation of the tether during deployment is visible, and essential effects are not noticeable. But in Fig. 5, in which the dependence of the relative angular velocity of the tether in orbital basis on the number of program turns is represented, it is visible that in the range $n_{rot} \in (45, 60)$ and in the range $n_{rot} > 80$ in the law of program change of the tether length, there are parts where it is necessary to increase the angular velocity of the tether, i.e., to reduce its length. As it was already noted, it will lead to complication of the mechanism of the system of deployment. Note that the range $n_{rot} > 80$, where cable length reduction is required, is also present in Fig. 2. However, due to the chosen viewing angle, the decrease in the coordinates L (representing length vs. time) on this interval is nearly indistinguishable from adjacent ranges.

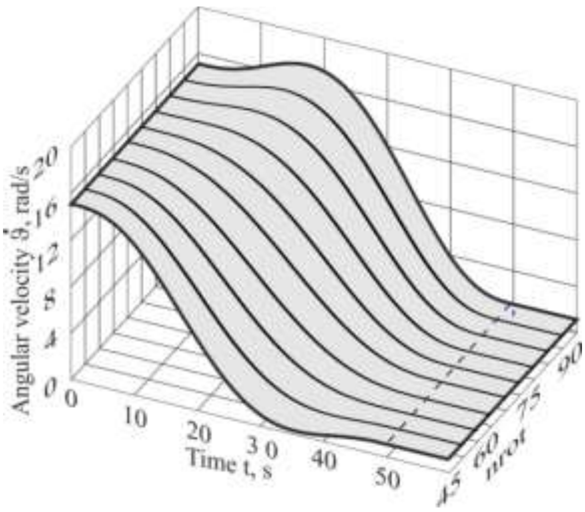

Fig. 5

Program change in time of the tether angular acceleration $\ddot{\vartheta}(t)$ depending on the number of its turns during deployment n_{rot} is shown in Fig. 6

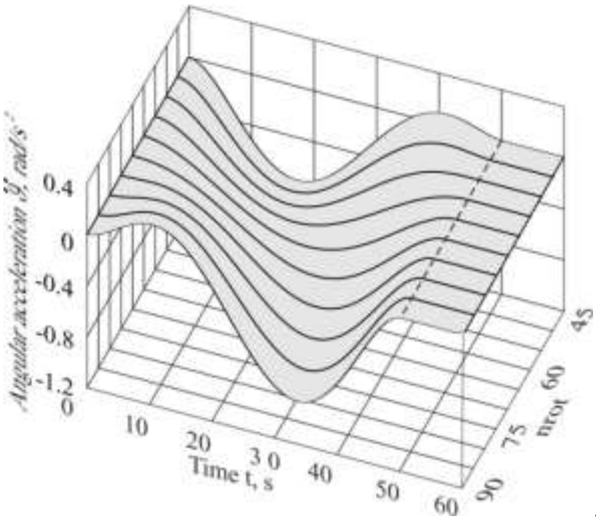

Fig. 6

In this figure it is visible that the number of turns in the program motion is less, and the maximum delay of rotation is closer to the beginning of deployment. Here it is also possible to note that in the range of the modes of the program motion acceptable for implementation ($n_{rot} \in (60,80)$) the range of change of the angular acceleration is narrower than at the edges of the considered area of values of the parameter n_{rot}.

Having constructed the program law of change of the tether length in time, it is possible to construct the program law of change of the tether tension T_{teth} in time. For this purpose we use one of the equations of motion of a tether of variable length in spherical coordinates [4] on the condition of the motion of the tether in the orbit plane. As a result, it is possible to write down:

$$T_{teth} = m/2\{-\ddot{L} + L[\left(\omega^{or} + \dot{\vartheta}\right)^2 \cos\varphi + \dot{\varphi}^2 + 3(\omega^{or})^2 \cos^2\vartheta \cos^2\varphi - (\omega^{or})^2 = m/2\{-\ddot{L} + L[\left(\omega^{or} + \dot{\vartheta}\right)^2 + 3(\omega^{or})^2 \cos^2\vartheta - (\omega^{or})^2 . \quad (20)$$

In Figure 7, the change in time of program values of the tether tension is shown in the mode of deployment depending on the chosen number of turns of the tether.

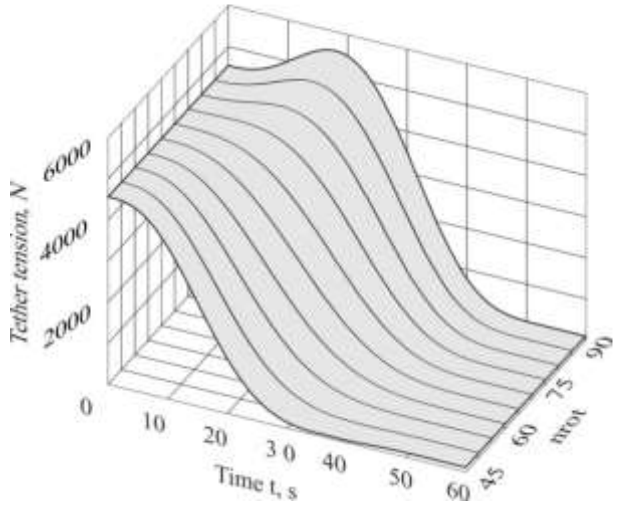

Fig. 7

Both in the initial and in the final instants of the deployment, the force of the tether tension does not depend on the total number of turns. Nevertheless, the nature of the change of the tension during deployment depends on value n_{rot} essentially. In the range of values $n_{rot} \in (80, 90)$ time intervals are visible on which the maximum tension force considerably exceeds the initial tension. It occurs where, according to Fig. 5, at the same values n_{rot} there is a significant increase in angular velocity of the tether. Respectively, significant decrease in the force of tension on an interval of time in the neighborhood of a point $t = 35$ s at $n_{rot} \in (45, 60)$ happens where, in Fig. 5, the considerable decrease in angular velocity in comparison with its value at the end of deployment is visible.

Lengthening of the tether during deployment is shown in Fig. 8.

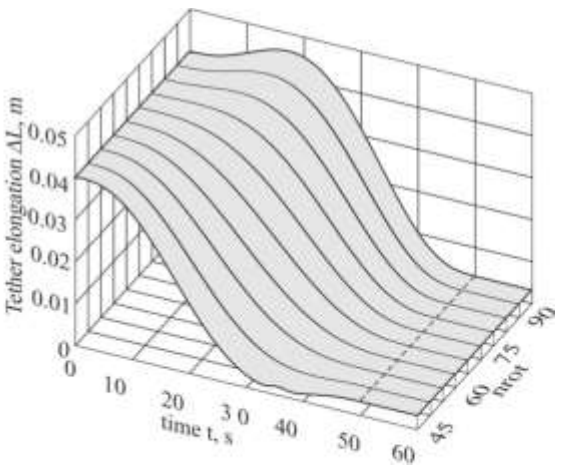

Fig. 8

The surface shape $\Delta L(n_{\mathrm{rot}}, t)$ at first sight reminds the surface shape $T_{teth}(n_{\mathrm{rot}}, t)$, nevertheless, here the similarity is rather mirror-like. This surface is much closer in shape to the surface $\dot{\vartheta}(n_{\mathrm{rot}}, t)$.

Having analyzed the graphic information connected with studying the influence of the parameter n_{rot} on the nature of the feedforward control law for the tether length during its deployment, we will note that the choice of value of this parameter does not allow us to obtain the set size of the final length of the tether. It follows from physical reasons that the final length of the rotating tether depends on its initial angular momentum and the specified final angular velocity of the tether in the orbital frame of reference. The initial relative tether angular momentum in the orbital frame of reference is

$$\vec{K}^C = \{0, 0, 2\, m\, r_0^{\,2} \dot{\vartheta}_0\}, \tag{21}$$

i.e., with constant end masses, the parameters r_0 and ϑ_0 equally influence the value of the angular moment and the size of the final length of the tether.

Further, we will turn to consideration of the influence of the parameter r_F on the control law and behavior of the tether. The nature of the change of the tether length at change of size r_0 in the range $r_0 \in [2, 2.9]$ is shown in Fig. 9. Other parameters at the same time have their basic values.

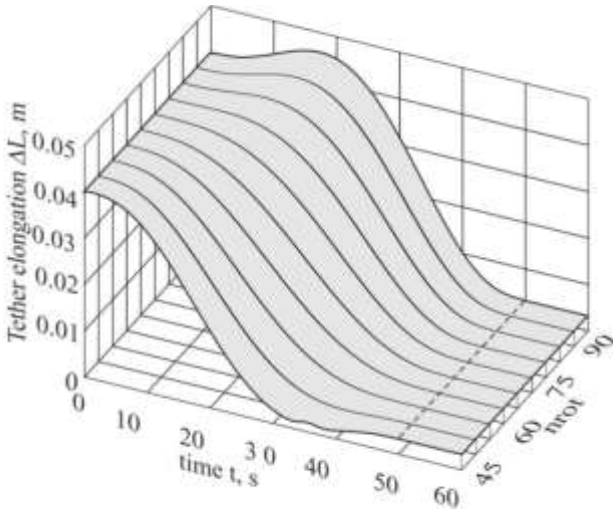

Fig. 9

In this figure it is visible that during the uniform increase of the initial size r_0 from 2.0 to 2.9 m, the radius of the deployed tether r_F evenly increases from 15.48 m up to 22.45 m. Here it should be noted that the choice of a way of increasing the initial angular momentum for an increase in the tether's final length, taking into account the tensile strength of the cable, will unambiguously lead to an increase in the initial radius of the tether, but not its initial angular velocity. It is easy to show, for example, that a fourfold increase in the module of the angular momentum due to an increase r_F twice, centrifugal force $F_C = mr_0(\dot{\vartheta}_0)^2$ will increase twice. To reach the same value of the angular momentum due to an increase in the initial angular velocity of rotation, it needs to be increased four times. At the same time, centrifugal force will increase eight times. It will demand an increase in tensile strength and of the cable mass.

The described result can be used for obtaining the specified final length of the tether, applying the method of linear interpolation.

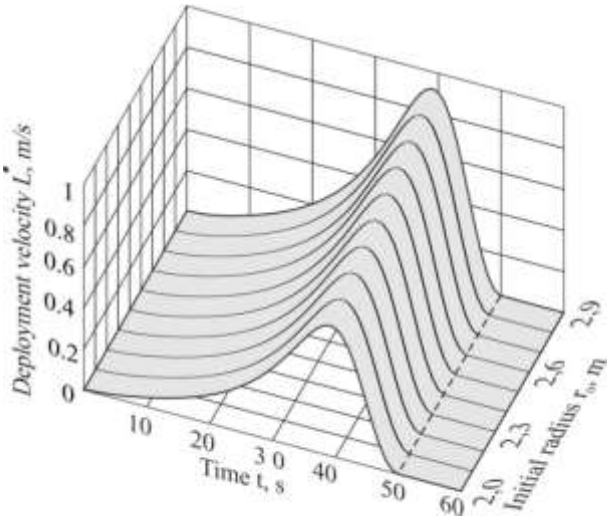

Fig. 10

In Figure 10 the velocity of change of the tether length during deployment is shown. The shown curves have smooth character. The less initial size of the radius r_0 , the less maximum value of velocity of the tether deployment.

Change of the initial value r_0 at constant value of the parameter n_{rot} does not influence laws of change of the pitch angle and all its time derivatives. In general, at correctly chosen value of the parameter n_{rot} the modes of deployment of the tether can be theoretically realized at any value n_{rot} .

Let's consider what impact the gravitational field has on the studied process. The plot of the gravitational torque acting on the tether during deployment at basic values of all parameters is provided in Fig. 11.

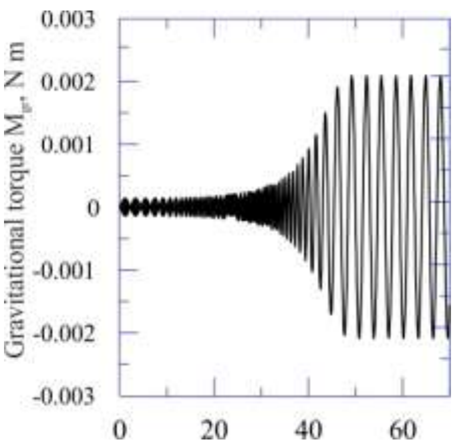

Fig. 11

Here it is visible that the value of the torque changes periodically, being synchronized with the frequency of rotation of the tether and with its length. In the beginning, the amplitude of the torque grows slowly, as well as the tether length in

Fig. 9, and quickly increases at the end, reaching a constant value at the instant $t = T_F$. In general, the maximum value of the amplitude of the gravitational torque is insignificant because of the small sizes of the tether. Therefore, this torque cannot have a significant effect on the nature of the deployment process of the tether. Nevertheless, in Fig. 12 the influence of this torque on the angular velocity of the deployed tether is shown upon termination of deployment.

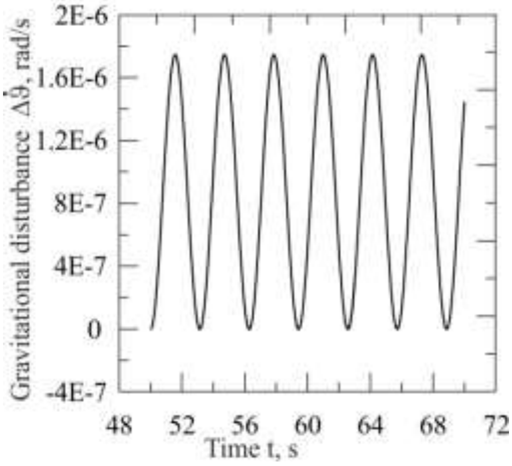

Fig. 12

Here it is visible that the angular velocity of the deployed tether rotating around the orbit binormal periodically changes with a frequency twice greater than the frequency of the tether rotation. Such perturbations of the angular velocity from the practical point of view do not matter. Naturally, the same perturbations may also be shown in the change of the tension force of the tether. The amplitude of fluctuations of the tether tension T_{teth} is about 0.0002 N for the considered case that taking into account the accepted rigidity of the cable on stretching has the negligible effect from the practical point of view.

Let us carry out a numerical simulation of the use of the constructed feedforward control law $L(t)$ for deployment of the tether to the condition of rotation with the specified length of 16 m and angular velocity of rotation in orbital basis 1 rad/s. Linear interpolation of the results of the numerical experiment represented in Fig. 9 shows that for obtaining the tether's final length $L_F =$ 16 m the initial length of the tether L_0 has to be equal to 4.1304 m. For numerical simulation of the deployment, we will use the full system of the equation of motion of the tether of variable length in a circular orbit in spherical coordinates [0] which describes the spatial motion of a tether. We will adopt the initial position of the tether in the orbital plane.

In Figure 13, the trajectory of the end body, upper at the initial instant is shown during deployment. The trajectory of the lower body is shifted in a phase on the angle π . As well as it was specified, the final length of the tether is 16 m.

Now it is necessary to turn to the elastic properties of the cable. Apparently in Fig. 8, at the initial instant when the tether tension in the majority of the considered cases is as much as possible, lengthening of the cable ΔL can reach several centimeters. Therefore, the mechanism of deployment has to have an opportunity to consider adjustment for lengthening of the cable at deployment of the cable from one of the bodies according to the obtained the length control law.

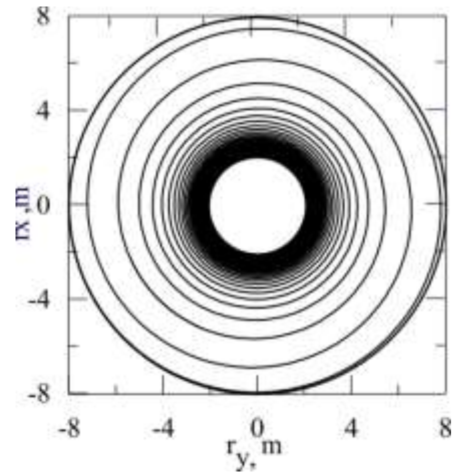

Fig. 13

Conclusion. Summarizing the results, it can be noted that a new approach is presented here to solve the problem of deploying a two body space tether under uniform rotation in a circular orbit, theoretically sound and numerically validated for length control. This approach relies on programmatic control of tether length, acknowledging the impossibility of directly controlling the tether pitch angle. The tether length control law is derived by imposing constraints on the motion law of the tether concerning pitch motion. These constraints arise from physical reasons and do not contradict the nature of the studied process. Consequently, the number of degrees of freedom decreases by one, and the problem obtains a simple solution. In control theory terms, the method for creating the programmed length control is developed, allowing deployment of the two tethered bodies under uniform rotation with a specified size and rotation frequency without triggering longitudinal fluctuations. The resulting control law is very smooth in time and lacks features that could cause such fluctuations.

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