



МЕТОДИ АНАЛІЗУ ТА УПРАВЛІННЯ СИСТЕМАМИ В УМОВАХ РИЗИКУ І НЕВИЗНАЧЕНОСТІ

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THE DYNAMIC PROGRAMMING METHOD APPLICATION TO THE SOLUTION OF ONE FUZZY SALESMAN PROBLEM

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Abstract. A method for solving the traveling salesman problem is proposed, utilizing dynamic programming to determine the shortest duration route, considering the fuzzy representation of travel time between individual points. Approaches for the approximation of fuzzy values, arithmetic operations, and methods for ordering fuzzy numbers are presented. The problem formulation with trapezoidal fuzzy numbers is considered. A representation form of such fuzzy numbers based on a Gaussian-like approach is proposed. Methods of the state space tree and dynamic programming are employed. The proposed technique is illustrated with an example.

Keywords: fuzzy traveling salesman problem, trapezoidal fuzzy numbers, defuzzification, modelling, optimization, dynamic programming.

INTRODUCTION

The problems of modern logistics have certain features that lead to difficulties in solving various logistical problems, some of which can be solved due to the management department work. Other tasks require the analysis and optimization of logistics operations, including planning, coordination and control of the movement and storage of goods, services and information. The involvement of mathematical approaches for solving logistics problems is gaining widespread implementation, the specific content of which depends on the nature of the problem and the available data. Sometimes it is possible to find unconventional methods of solving known problems, one of which is the traveling salesman's problem.

The traveling salesman's problem was first formulated by the Irish mathematician V.R. Hamilton in the 19th century. The problem describes the need to draw up a route within a given set of interconnected points (cities) that form the transport network of a specific region. A traveling salesman needs to make a route according to which he has to visit all the cities in the network, satisfying the condition that the total distance of the route or the total time of the travelling are minimal. The peculiarity of the problem is that the route must contain all the points specified in the problem, and each of the points must be visited no more than once.

The salesman's problem is a combinatorial problem that can be solved using mathematical programming methods. For certainty, the cities can be numbered as

$(1, 2, 3, \dots, n)$, then the route of the traveling salesman can be described by a cyclic sequence of city numbers $t = (j_1, j_2, \dots, j_n, j_1)$, thereby j_1, \dots, j_n are different numbers. Any permutation of numbers presented in this form represents a possible problem solution, and therefore there are $(n - 1)!$ possible ways to construct its route. The traveling salesman's problem is to choose the route that is optimal in terms of trip cost or trip duration, which satisfies some given constraints.

In the real world, the concept of the duration or cost of travel between individual points of the transport network cannot have fixed values, they should be determined approximately. Often these values affected by the influence of subjective analysis regarding estimates of time periods or the cost of moving along sections of the route. This leads to the necessity of considering uncertainty and its formalization based on various methodologies. One of the approaches used in this case is to involve the methods of the theory of fuzzy sets and numbers.

In this case, the objective function coefficients and constraints, which describe the uncertainty, make it possible to formalize the inaccuracy of the numerical characteristics of the optimization problem. Optimization problem statements with inherently fuzzy data are fuzzy optimization problems, one of which is the fuzzy traveling salesman problem (FTSP).

Traditional problems of solving optimization tasks become significantly more complicated in cases where imprecise parameters are considered, which limits the possibility of applying known approaches to problem-solving. The need to find optimal solutions, the importance of effectively solving the traveling salesman's problem under conditions of uncertainty and inaccuracy prompt the search for different methods and algorithms for solving the problem in question.

The fuzzy sets theory, as one of the approaches to the formalization of uncertainty, was proposed by L. Zade [1]. Its application for solving applied optimization problems is reflected in many studies [2–8]. Among the publications devoted to the traveling salesman problem, we highlight the research [2], which discusses the ways of “solving the fuzzy traveling salesman problem with various membership functions”. The paper [3] proposed the concept of decision-making in a fuzzy environment. The study [4] is devoted to methods of solving fuzzy linear programming problems taking into account several objective functions. On the other hand, many different methods have been proposed for solving the classic traveling salesman problem, as a combinatorial problem, based on greedy and heuristic approaches that allow finding local optimal solutions (for example, [5, 6]). Therefore, the combination of these methods with the possibility of using fuzzy numbers to account for uncertainty in optimization tasks presents both theoretical and practical interest.

The goal of the this article is to develop an algorithm for solving the traveling salesman problem with a fuzzy defined travel times in a transport network. The object of the study is the process of finding the optimal route for the fuzzy traveling salesman problem with a minimum travel time. The subject of the study is the development of an effective algorithm for solving the fuzzy traveling salesman problem, provided that the time of movement between individual cities along the route is presented in the form of trapezoidal fuzzy numbers. The approach is based on the application of the dynamic programming method.

GENERAL CONCEPTS OF FUZZY SETS AND FUZZY NUMBERS

Definition 1. [9] A fuzzy set \tilde{A} of a universal set X is the set of pairs $\tilde{A} = \{(\mu_{\tilde{A}}(x), x)\}$, where $\mu_{\tilde{A}} : X \rightarrow [0,1]$ is a mapping of the set X into a unit interval $[0,1]$ and is called a membership function of the fuzzy set \tilde{A} .

The membership function value $\mu_{\tilde{A}}(x)$ for an element $x \in X$ is called the membership degree. The interpretation of the membership degree $\mu_{\tilde{A}}(x)$ is a subjective measure of degree of an element $x \in X$ conformity to the concept, the content of which is formalized by a fuzzy set \tilde{A} .

Definition 2. A fuzzy set \tilde{A} is called convex [9] if

$$\mu_{\tilde{A}}(\lambda x + (1-\lambda)y) \geq \min(\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)) \text{ for any arbitrary } x, y \in X, \lambda \in (0,1).$$

As a universal set X let us consider the set of real numbers, i.e. $X = R^1$.

Definition 3.[9] A fuzzy set \tilde{A} defined on the set of real numbers R^1 is a fuzzy number if the following conditions are met:

- 1) the set \tilde{A} is convex;
- 2) the set \tilde{A} is normal, in this context, there exists $x \in R$ such that $\mu_{\tilde{A}}(x) = 1$;
- 3) the membership function $\mu_{\tilde{A}}(x)$ is upper semicontinuous;
- 4) support of a fuzzy number $\text{sup } p(\tilde{A})$ is a subset of the universal set R^1 .

Definition 4. [9], Fig. 1. A trapezoidal fuzzy number \tilde{A} is an ordered tuple of four real numbers (a_1, a_2, a_3, a_4) , $a_1 \leq a_2 \leq a_3 \leq a_4$ that defines the membership function $\mu_{\tilde{A}}(x)$ to the form:

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1}, & \text{when } a_1 \leq x \leq a_2 \\ 1, & \text{when } a_2 \leq x \leq a_3 \\ \frac{a_4-x}{a_4-a_3}, & \text{when } a_3 \leq x \leq a_4 \end{cases}$$

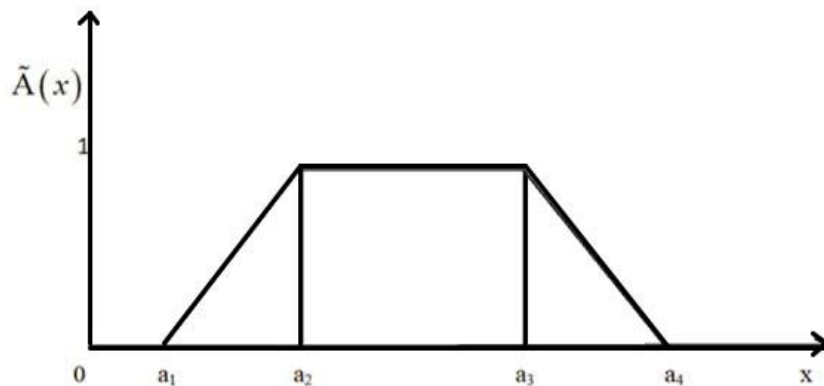


Fig. 1. Trapezoidal fuzzy number

If the approach based on the Gaussian-like distribution with corresponding characteristics is applied to the definition of a fuzzy number, then in the generalized case, the trapezoidal fuzzy number can be represented in a slightly different form:

$$\tilde{A} = (a_1, a_2, a_3, a_4) = ([a_2, a_3], \alpha, \beta) = (m, w, \alpha, \beta), \quad (1)$$

where the midpoint $m = \frac{a_2 + a_3}{2}$ and half-width of the plateau $w = \frac{a_3 - a_2}{2}$ are used, and the coefficients $\alpha = a_2 - a_1$ and $\beta = a_4 - a_3$ determine the width of left and right boundaries of the fuzzy number $\tilde{A} = (a_1, a_2, a_3, a_4)$, respectively.

ARITHMETIC OPERATIONS WITH TRAPEZOIDAL FUZZY NUMBERS

Let's define operations with fuzzy numbers based on the description above. As the midpoint, the usual arithmetic mean value of the plateau boundaries is taken, the left and right distributions are considered according to the lattice rule, according to which for arbitrary real numbers a, b we put $a \cup b = \max\{a, b\}$ and $a \cap b = \min\{a, b\}$.

For arbitrary trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$, that written in the form $\tilde{A} = (m(\tilde{A}), w(\tilde{A}), \alpha_1, \beta_1)$ and $\tilde{B} = (m(\tilde{B}), w(\tilde{B}), \alpha_2, \beta_2)$, we define the following operations of addition, subtraction, multiplication, and division, which in the general case are denoted by the symbol \circ :

$$\tilde{A} \circ \tilde{B} = (m(\tilde{A} \circ \tilde{B}), w(\tilde{A} \circ \tilde{B}), \alpha(\tilde{A} \circ \tilde{B}), \beta(\tilde{A} \circ \tilde{B})).$$

Finally we have

$$\begin{aligned} \tilde{A} + \tilde{B} &= (m(\tilde{A}) + m(\tilde{B}), w(\tilde{A}) + w(\tilde{B}), \alpha_1 + \alpha_2, \beta_1 + \beta_2), \\ \tilde{A} - \tilde{B} &= (m(\tilde{A}) - m(\tilde{B}), w(\tilde{A}) + w(\tilde{B}), \alpha_1 + \beta_2, \beta_1 + \alpha_2), \\ \tilde{A} * \tilde{B} &= (a_2 m(\tilde{B}) + b_3 w(\tilde{A}), b_3 m(\tilde{A}) - a_2 m(\tilde{B}), b_1 \alpha_1 + a_2 \alpha_2, b_3 \beta_1 + a_4 \beta_2), \\ \tilde{A} \div \tilde{B} &= (m(\tilde{A}) / b_3 + w(\tilde{B}) a_3 / b_2 b_3, w(\tilde{B}) a_3 / b_2 b_3 + w(\tilde{A}) / b_3, \\ &\quad \alpha_1 / b_3 + \beta_2 a_1 / b_3 b_4, \beta_1 / b_1 + \alpha_2 a_3 / b_1 b_2) \end{aligned}$$

ORDERING OF TRAPEZOIDAL FUZZY NUMBERS

For the implementation of comparison operations and ranking of fuzzy numbers, it is natural to use a method based on the median average value. In other words, if a ranking function $\mathfrak{R}: F(R) \rightarrow R$ with a median average value of the form

$$\mathfrak{R}(\tilde{A}) = \left[\left(\frac{a_2 + a_3}{2} \right) + \left(\frac{\beta - \alpha}{4} \right) \right]$$

is defined for each $\tilde{A} = (a_1, a_2, a_3, a_4) \in F(R)$,

then for any two trapezoidal fuzzy numbers $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ we have the following possible comparison options:

- $\tilde{A} \succ \tilde{B}$ then and only then, if $\mathfrak{R}(\tilde{A}) > \mathfrak{R}(\tilde{B})$;
- $\tilde{A} \prec \tilde{B}$ then and only then, if $\mathfrak{R}(\tilde{A}) < \mathfrak{R}(\tilde{B})$;
- $\tilde{A} \approx \tilde{B}$ then and only then, if $\mathfrak{R}(\tilde{A}) = \mathfrak{R}(\tilde{B})$.

THE PROCEDURE OF DEFUZZIFICATION OF TRAPEZOIDAL FUZZY NUMBERS

The processes of handling fuzzy numbers involve a defuzzification stage – converting a fuzzy result into a precise (numerical) value. This is an important step in the methodology of applying a fuzzy approach, especially in tasks of fuzzy control and fuzzy business logic, where it is necessary to transform fuzzy solutions into specific events or numerical values. There are various defuzzification methods, among which the most commons are: the method of center of gravity (CoG) or centroid and the method of mean maximum (MoM). To conduct a comparison of the research results, we will use the center of gravity method. In this method, the defuzzification point is calculated as the center of gravity of the fuzzy set. For a discrete set, the formula looks like this:

$$CoG = \frac{\sum_{i=1}^n \mu(x_i) \cdot x_i}{\sum_{i=1}^n \mu(x_i)},$$

where x_i are the points in the result space, and $\mu(x_i)$ is the membership degree for each point. In the case of a continuous distribution, the formula looks like this:

$$CoG = \frac{\int_{a_1}^{a_4} x \cdot \mu(x) dx}{\int_{a_1}^{a_4} \mu(x) dx}.$$

MATHEMATICAL FORMULATION OF FUZZY TRAVELING SALESMAN PROBLEM

Let's return to the traveling salesman problem. If the route duration of the salesman is considered as the criterion of the problem, the task is clearly defined. A different

situation arises when searching for the fastest route considering the fuzzy formalization of travel time between individual points in the city network. In this case, we have a fuzzy traveling salesman problem.

In the case of the fuzzy traveling salesman problem stating, it is necessary to find a cyclic permutation of the city numbers that the salesman needs to visit, according to which the time cost will be minimal, considering the standard constraint, that each point must be visited no more than once and return to the starting point. The mathematical formulation of the fuzzy traveling salesman problem can be written as follows: it is necessary to minimize the objective function, taking into account the above-mentioned method of comparing fuzzy numbers:

$$\sum_{i=1}^n \sum_{j=1}^n \tilde{t}_{ij} x_{ij},$$

where the time costs for moving between points are given in the form of a matrix with elements as trapezoidal fuzzy numbers, and the possible paths of movement between cities are determined by the subject to the fulfillment of the constraints:

$$\begin{aligned} \sum_{i=1}^n x_{ij} &= 1 \text{ for all } j = 1, 2, \dots, n, \\ \sum_{j=1}^n x_{ij} &= 1 \text{ for all } i = 1, 2, \dots, n, \\ v_i - v_j + nx_{ij} &\leq n - 1, \quad 1 \leq i \neq j \leq n, \\ x_{ij} &= 0 \text{ or } 1 \text{ for all } i, j = 1, 2, \dots, n. \end{aligned}$$

For software implementation in the matrix T , the elements \tilde{t}_{ii} must be set with large positive numbers in order to obtain values in solution $x_{ii} = 0$ for all $i = 1, 2, \dots, n$.

DYNAMIC PROGRAMMING ALGORITHM FOR SOLVING THE FUZZY TRAVELING SALESMAN PROBLEM

Let us consider the traveling salesman problem with fuzzy defined travel durations between individual cities, which are presented in the form of trapezoidal fuzzy numbers. One of the approaches to solving traveling salesman problems is based on the use of the dynamic programming method, the main idea of which is to divide the input task into subtasks and memorize solutions to avoid repeated calculations. This allows constructing an optimal solution for the large problem based on optimal solutions connections of individual subproblems or based on the solutions of some recursive sequence of nested calculations.

The essence of the main principle of dynamic programming is as follows. A multi-step (multi-stage) process of searching the optimization problem solution is considered, an arbitrary step of which can be described in the form $[\alpha, P(\alpha, x)]$, where α is the state that characterizes the current content of the problem, the function $P(\alpha, x)$ defines the state transformation procedure (transition function), and x is the solution vector. Obviously, the transition function depends on the state variable α and the vector of independent variables x .

Assume that at each step of the search process there is an opportunity to choose a solution vector x from some feasible set. Let x_n is the vector of variables chosen at the n -th step. Then we get the sequence of changes in the state of the problem in the form: $\alpha = \alpha_0$ (initial state); $\alpha_n = P(\alpha_{n-1}, x_{n-1})$, $n = 2, 3, \dots$.

Let us consider processes in which the variable vectors x_n , $n = 2, 3, \dots$, are chosen in such a way as to maximize a certain predetermined criterion function $R(\alpha_1, \alpha_2, \dots, \alpha_N, x_1, x_2, \dots, x_N)$. We will call the solution to be optimal if it corresponds the maximum of the given criterion function.

Considering the generalized nature of the accepted definition of the criterion function R , we should note that the solution vectors x_n , $n = 2, 3, \dots$, must depend on the current state of the system, as well as on previous and subsequent states and solutions. However, there are criterion functions of a special structure in which the solution depends only on the current state. In this special but very important case, the optimal strategy is determined by the Bellman's optimality principle [10], which forms the basis of the dynamic programming method: *an optimal strategy has the property that, whatever the initial states and the first solution, the subsequent solutions constitute an optimal strategy in relation to the initial state obtained as a result of the first solution.*

Let's consider the problem of maximizing the function of a special structure

$$R(\alpha_1, \alpha_2, \dots, \alpha_N, x_1, x_2, \dots, x_N) = \sum_{n=1}^N g_n(\alpha_n, x_n), \quad (2)$$

where $g_n(\alpha, x)$ – the solution selection function at the n -th step, $n = 1, 2, \dots, N$.

Suppose that $f_n(\alpha)$ is the maximum value of the criterion function R , starting from the state α , in steps $n = 2, 3, \dots, N$, respectively. Then Bellman's principle of optimality can be written in the following form:

$$\begin{aligned} f_N(\alpha) &= \max_x g_N(\alpha, x), \\ f_n(\alpha) &= \max_x \{g_n(\alpha, x) + f_{n+1}(P(\alpha, x))\}, \\ n &\leq N - 1. \end{aligned} \quad (3)$$

The equations set (3) is called the Belman equation (the basic functional equation of dynamic programming method) [10].

One of the implementations of the dynamic programming method for solving the traveling salesman problem is the Held–Karp algorithm [11], which is known to have time complexity $O(n^2 2^n)$ and space complexity $O(n 2^n)$, which makes it much faster than a complete search, especially on relatively small sets of input data.

To formalize the approach for solving the traveling salesman problem on the set of cities in the network, let's define a graph $G(S, E)$, where S is the set of vertices (nodes), E is the set of edges.

Let $C(S, i)$ denote the duration of the shortest salesman path starting and finishing in i -th city (node) by the constraint of visiting each city in current subset \bar{S} of node set S only once, $\bar{S} \subset S$. Then the value $C(\bar{S}, i)$ can be calculated using the following formula:

$$C(\bar{S}, i) = \min_{j \in \bar{S}, j \neq i} (C(\bar{S} \setminus \{i\}, j) + t_{ij}),$$

where \bar{S} is a subset of all cities, except for some initial one, which determines the state of the problem; i and j – numbers of cities in the set \bar{S} ; t_{ij} – duration of time from city i to city j ; $C(\bar{S} \setminus \{i\}, j)$ – duration of the shortest path along which the traveling salesman visits all cities in \bar{S} , excepting i -th city, starting and finishing in the j -th city.

Just as in the case of the deterministic traveling salesman problem, for the fuzzy variant, this approach is implemented on the basis of the construction of a state space tree that contains all possible paths. If the matrix of the duration of movements for example, has the form

$$T = \begin{bmatrix} \infty & (0.5, 1, 2, 3, 5) & (1.5, 2, 2.5, 3, 5) & (2, 2.5, 3, 4) \\ (0.5, 1, 2, 3, 5) & \infty & (2, 3, 4, 5, 5) & (1.5, 2, 2.8, 3, 9) \\ (1.5, 2, 2.5, 3, 5) & (2, 3, 4, 5, 5) & \infty & (1, 2, 2.5, 3) \\ (2, 2.5, 3, 4) & (1.5, 2, 2.8, 3, 9) & (1, 2, 2.5, 3) & \infty \end{bmatrix},$$

then the state space tree in this case will have the form shown in Fig. 2

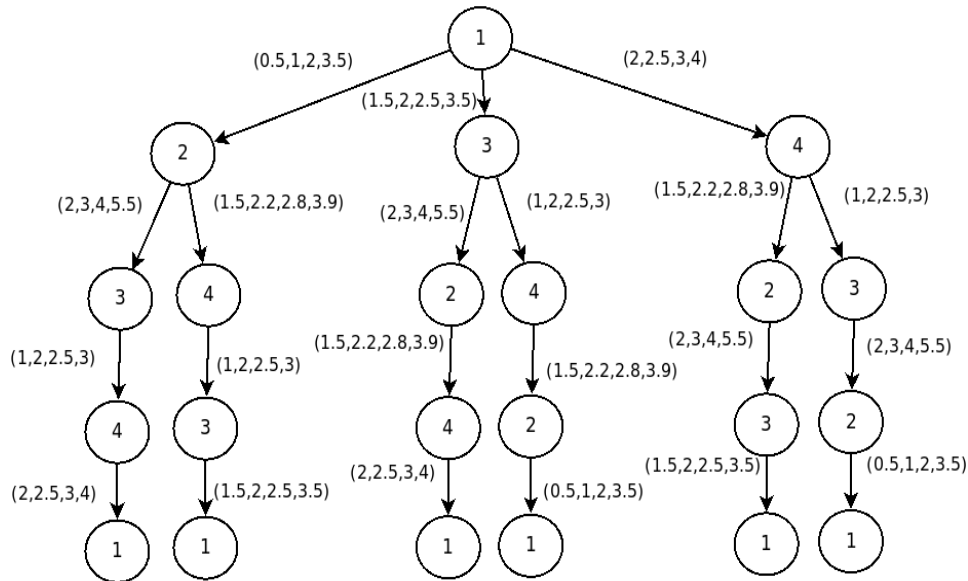


Fig. 2. State space tree for the traveling salesman problem with the displacement matrix (3)

Then the following algorithm can be applied to solve the fuzzy salesman problem using the dynamic programming method:

Step 1. Let's transform the duration of each movement specified in the matrix T in a modified form (1) $\tilde{A} = (a_1, a_2, a_3, a_4) = ([a_2, a_3], \alpha, \beta) = (m, w, \alpha, \beta)$.

Step 2. Let's build a tree of the state space with any arbitrary node as the main one, and the other nodes are considered as branches. This diagram will contain all possible routes for travel.

Step 3. Let's apply the formulated algorithm based on the implementation of the dynamic programming principle (2) to find the optimal path under the conditions of performing arithmetic operations on fuzzy numbers.

Step 4. We will determine the shortest route based on the defuzzification of the obtained fuzzy values, which, according to the problem constraints, will pass through each city only once and return to the initial node.

NUMERICAL EXPERIMENT RESULTS

As a first example, consider the deterministic traveling salesman problem with a given set of travel durations on a network of 11 cities, which was presented in [12]. The corresponding NRAF network in this case has the form (Fig. 3).

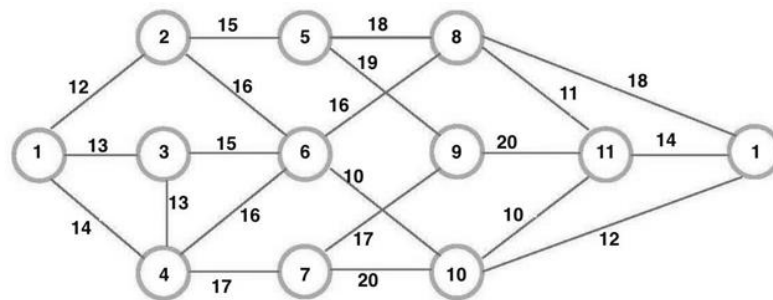


Fig. 3. An example of a network of cities with travel time for the specific traveling salesman problem

The optimal route calculated by the dynamic programming method and full search in both cases is 156 time units, which is implemented by the following sequence of visits to cities

$$1 \rightarrow 2 \rightarrow 6 \rightarrow 10 \rightarrow 11 \rightarrow 8 \rightarrow 5 \rightarrow 9 \rightarrow 7 \rightarrow 4 \rightarrow 3 \rightarrow 1.$$

Numerical calculations were carried out to evaluate and compare the effectiveness of algorithms. For the given network (non-existent paths in the corresponding matrix T were coded with a time value of 1000), the average calculation time based on a complete search was approximately 1 seconds, and for the dynamic programming method – 6 ms.

In the case of using fuzzy values of the duration of movement between nodes for the traveling salesman problem with matrix T (3), the corresponding indicators for both methods practically do not differ.

As part of a more complex numerical experiment, sets of cities on the plane were generated from points, corresponding matrices of time durations of movements were obtained, for which fuzzy representations were randomly formed. The given calculations (Table) demonstrate the effectiveness of calculations by the full-search method and the dynamic programming method.

Indicators of time costs for calculating the optimal path

City quantity	Complete search, s	Dynamic programming method, ms
11	0.948	5.484
12	10.73	7.976
13	133.75	14.15
14	1910.7	29.45
15	28851	70.38

CONCLUSIONS

An approach for constructing a solution to the fuzzy traveling salesman problem is proposed, provided that the time of movement between individual cities along the route is presented in the form of trapezoidal fuzzy numbers. The approach is based on the application of the dynamic programming method, which made it possible to formulate a practical algorithm for solving the fuzzy traveling salesman problem. Approximation methods for fuzzy numbers, arithmetic operations and ways to arrange fuzzy numbers are presented. A method for transforming trapezoidal fuzzy numbers based on the Gaussian approach is proposed. The implementation of the algorithm uses the technique of building a state space tree using an original computing scheme that implements the principle of dynamic programming in the form of an algorithm with the considered practical implementation of arithmetic operations on fuzzy numbers. The proposed methodology has been illustrated with examples.

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ЗАСТОСУВАННЯ МЕТОДУ ДИНАМІЧНОГО ПРОГРАМУВАННЯ ДО РОЗВ'ЯЗАННЯ ОДНІЇ НЕЧІТКОЇ ЗАДАЧІ КОМІВОЯЖЕРУ/ Є.В. Івохін, К.Е. Юштин, Л.Т. Аджубей

Анотація. Запропоновано підхід для розв'язування задачі комівояжера на основі застосування методу динамічного програмування з метою визначення найкоротшого за тривалістю маршруту з урахуванням нечітко заданого часу на переміщення між окремими пунктами. Наведено способи апроксимації нечітких величин, арифметичних дій та способів впорядкування нечітких чисел. Розглянуто постановку задачі з трапецієподібними нечіткими числами. Запропоновано форму подання таких нечітких чисел на основі гаусоподібного підходу. Використано методи дерева простору станів і динамічного програмування. Запропоновану методику проілюстровано прикладом.

Ключові слова: нечітка задача комівояжера, трапецієподібні нечіткі числа, дефазифікація, моделювання, оптимізація, динамічне програмування.