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RANDOM ANGLE GENERATOR IN THE ATMOSPHERIC MONITORING SYSTEM OF ENERGY FACILITIES

Abstract. *The complication of maintaining the power balance in the energy system under martial law conditions, ensuring the normal operating modes of power facility equipment, and maintaining the required quality of fossil energy resources used, among other factors, leads to an increase in harmful emissions into the atmosphere. Therefore, the development and improvement of environmental monitoring tools for energy facilities has become particularly relevant. In environmental monitoring systems, primary measurement information is generated by a system of sensors located around sources of pollutant emissions. One of the important tasks in the analysis of experimental data is the estimation of the distribution of pollutant concentrations in the vicinity of energy facilities using polar coordinates. To address this problem, modules for statistical analysis of circular data are included in the algorithmic and software support of monitoring systems. The task of verifying such modules requires the development of random angle generators capable of producing test samples of random angles with complex probability distributions. The article considers the methodological basis for solving problems of computer experiments with random angles, substantiates the use of multimodal probability distributions in problems of circular data analysis for atmospheric monitoring, proposes a random angle generator, and performs its verification using the example of generating a sample with a bimodal distribution. To evaluate the adequacy of the theoretical and empirical probability distributions of the random angle, the Kolmogorov criterion is used. The developed generator makes it possible to obtain samples of random angles with distributions defined analytically, by a table or graph (histogram), and can be used for testing algorithmic and software support in systems for statistical analysis of circular data.*

Keywords: random angle generator, computer experiment, trigonometric moments method, bimodal distributions of random angles.

1. Introduction

In Ukraine, energy facilities are among the potentially hazardous sources of atmospheric pollution [1, 2]. In recent years, under the conditions of martial law, the issues of environmental safety and environmental monitoring of energy facilities have become more acute and require special attention due to the increasing difficulty of maintaining the power balance in the energy system, the complexity of ensuring normal operating modes of energy facility equipment, and the need to maintain the appropriate quality of fossil energy resources, among other factors. This leads to an increase in harmful pollutants in the combustion products of thermal power plants. Therefore, the development and improvement of environmental monitoring tools for energy facilities has become particularly relevant. A number of publications are devoted to various theoretical and practical issues related to the development and implementation of environmental monitoring systems for energy facilities in Ukraine, including [3–10].

An important component of environmental monitoring systems for energy facilities is the subsystem for analyzing the spatial distribution of emissions of harmful substances into the atmosphere generated during the combustion of fossil fuels and the operation of auxiliary equipment at energy facilities. Such analysis makes it possible not only to determine the directions most vulnerable from the standpoint of environmental safety with respect to the spread of harmful substances in the air from the pollution source, but also to analyze the dynamics of their accumulation and concentration, to make forecasts regarding possible exceedances of permissible concentration limits of pollutants in areas adjacent to energy facilities, and to detect possible unauthorized emissions of harmful substances into the atmosphere.

The processes of formation and dispersion of harmful substances in the atmosphere are probabilistic in nature and depend on many factors, such as the quality of fossil fuels, their combustion regimes, wind patterns, and the operating modes of auxiliary equipment, among others. In environmental monitoring systems, primary measurement information about the state of the environment is generated by a system of sensors located around sources of pollutant emissions. One of the key issues in the analysis of experimental data obtained from such sensors is the estimation of the distribution of pollutant concentrations in the vicinity of energy facilities in polar coordinates. This problem can be solved using methods of mathematical statistics designed for the analysis of results of angular observations [11–13]. Therefore, the algorithmic and software support of environmental monitoring systems should include modules for the statistical analysis of experimental data with circular distributions. Certain aspects of the practical application of statistical methods for circular data analysis are considered in [14]; a probabilistic model of a random angle (RA) was developed in [15]; and studies in [16] investigated the application of the method of sample trigonometric moments to problems of approximating probability distributions of measurement results of signal phase shifts.

The application of statistical methods for the analysis of circular data in problems of processing sensor data governed by circular distributions has its own specific features and requires both additional analytical studies and the development of appropriate algorithmic and software support, as well as its verification. It should be noted that the concentration density distributions of harmful emissions around energy facilities, due to the influence of various factors, have a complex multimodal structure and are poorly approximated by known standard unimodal circular distributions. Therefore, solving the problem of verifying modules for the statistical analysis of circular data requires the development of RA generators capable of generating test sequences of RAs with complex multimodal probability distributions. In [8], the idea of using the inverse function method to generate unimodal circular data for testing phase meters was explored. This idea requires further development and confirmation of the possibility of its use for creating RA generators with multimodal probability distributions.

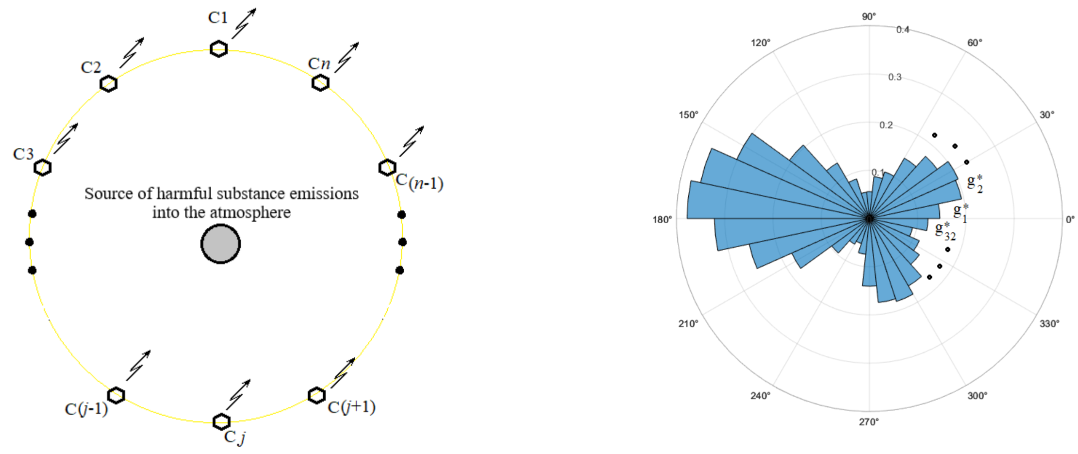
The aim of this paper is to develop a software-based random angle generator with multimodal distributions and to verify its performance.

2. Methodological Basis for Solving Computer Experiment Tasks with Random Angles in Atmospheric Monitoring Systems of Energy Facilities

2.1. Tasks and Theoretical Foundations of Computer Experiments for Generating RAs

When performing atmospheric monitoring around energy facilities with pollution sources, primary information about pollutants is obtained from a set of sensors C_1–C_n arranged in a circle (perimeter) at a considerable distance from the pollution source(s). An example of such a sensor system arrangement is shown in Fig. 1a.

The measurement information $\left(g_j,\ j=\overline{1,n}\right)[g]$, where $[g]$ is the dimensionality of the parameter g, from the sensors is transmitted to the data collection and processing unit. If this measurement information is normalized according to the expression $g_j^* = g_j \Big/ \sum_{j=1}^{n} g_j$, a new set $\left(g_j^*,\ j=\overline{1,n}\right)$ is obtained, for which $\sum_{j=1}^{1} g_j^* = 1$. This provides a formal basis to consider the elements of g_j^* as elements of a circular histogram (Fig. 1b) [11], obtained from the observation of random angles $\vartheta \in [0, 2\pi)$ (or $\vartheta \in [-\pi, \pi)$), and to apply circular probability distributions for their analysis [11–13]. These distributions have their specific measures of central tendency—circular mean, mode, and median—as well as measures of dispersion—circular variance and circular standard deviation.



a b

Fig. 1. Example of the arrangement of atmospheric monitoring sensors around a pollutant emission source (a) and the circular histogram in polar coordinates (b)

The inclusion of modules for processing circular data in the algorithmic and software support of air monitoring systems, in turn, requires informational support from computer experiments with RAs. A characteristic feature of RAs in atmospheric monitoring systems is that, due to various factors, the actual circular distributions of pollutant concentrations often exhibit a multimodal structure. This means that the corresponding probability density of RAs shows two or more peaks (modes), indicating data heterogeneity and the presence of different subgroups within the dataset. Such heterogeneity may, for example, arise from measurements obtained from different pollution sources or from changes in sampling conditions, such as shifts in wind direction.

A computer experiment for generating RAs, including those with multimodal probability distributions, is aimed at addressing the following main tasks: generating RA samples with a specified probability distribution, determining the probability distribution of RAs from sample data, and evaluating the adequacy of empirical and theoretical distributions (Fig. 2).

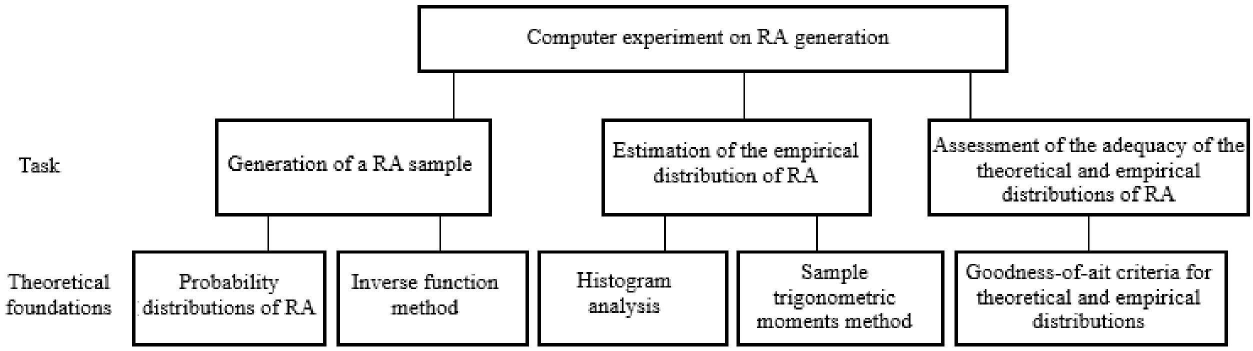


Fig. 2. Tasks and Theoretical Foundations of Computer Experiments for RA Generation

The cyclical nature of angular data makes it impossible to apply traditional methods used in the statistical analysis of linear random variables (RVs). Since methods of circular statistics have not yet gained widespread use in domestic scientific literature, the theoretical foundations necessary for understanding the proposed methodology of computer experiments with RAs are briefly outlined below.

2.2. Justification for Using the von Mises Circular Distribution in the Analysis of Circular Data for Atmospheric Monitoring

In the statistical analysis of circular observation data, various circular probability distributions are used, including the cardioid, von Mises, triangular, and wrapped distributions [11–13]. In [17], comprehensive information is provided on all known wrapped distributions to date – 45 distributions for continuous RAs and 10 distributions for discrete RAs. For each distribution, the main circular characteristics are indicated. In this work, we used the von Mises circular distribution, which has certain advantages compared to others, including a convenient analytical representation that simplifies statistical analysis and a satisfactory approximation of the wrapped normal distribution. The wrapped normal distribution has the property that the convolution of two wrapped normal distributions is itself a wrapped normal distribution [11–13]. These features of the von Mises distribution provided the basis for its use in constructing RA generators with multimodal distributions.

The von Mises probability density function for a random angle ϑ is defined as:

$$p_M(\theta \mid \mu, k) = \frac{1}{2\pi I_0(k)} exp\{k\,cos(\theta-\mu)\},\ \theta \in [0, 2\pi), |\mu| < \infty, k > 0, \tag{2.1}$$

where μ is the circular mean direction of the random RA, k is the concentration parameter of the distribution around μ, θ is the value of the RA, and $I_0(k)$ is the modified Bessel function of the first kind of order zero. The von Mises circular distribution is unimodal and symmetric with respect to the point $\mu(\mathrm{mod}2\pi)$ within the interval $(\mu-\pi, \mu+\pi)$, it has two inflection points. The cumulative distribution function (CDF) of the von Mises distribution does not have a simple analytical expression and is represented as an infinite series:

$$F_M(\theta \mid \mu, k) = \frac{1}{2\pi}\left[\theta + \frac{2}{I_0(k)}\sum_{q=1}^{\infty} I_q(k)\frac{\sin[q(\theta-\mu)]}{q}\right],\ \ |\mu| < \infty,\ k > 0. \tag{2.2}$$

A multimodal circular probability density will be defined as a weighted sum of independent RAs ϑ_s, $s = \overline{1, S}$ [18] with a unimodal von Mises circular distribution (2.1).

$$p(\theta) = \sum_{s=1}^{S} A_s \cdot p_M(\theta \mid \mu_s, k_s),\ \ A_s \in (0,1], \tag{2.3}$$

where S is the number of independent RAs, A_s is the weight coefficient, with $\sum_{s=1}^{S} A_s = 1$. A common case of a multimodal distribution is the bimodal distribution, which in general form can be represented as:

$$p(\theta) = A_1 \cdot p_M(\theta \mid \mu_1, k_1) + (1-A_1) \cdot p_M(\theta \mid \mu_2, k_2)\ . \tag{2.4}$$

The density of distribution (2.4) has five parameters: $\mu_1, \mu_2, k_1,\ k_2, A_1$. The parameter A_1 is also called the “mixing parameter,” and it represents the probability of occurrence of the RA ϑ_1 values in the mixture.

In bimodal distributions, the highest frequency of RA observations occurs at two distinct values. In the case of two unequal modes, the larger is referred to as the primary mode, and the smaller as the secondary mode. The antimode is the least frequent angle value between the modes. Bimodal distributions have a special property: unlike unimodal distributions, the mean value can provide a more reliable estimate of the sample than the median [19].

Bimodal distributions are encountered in the study of natural phenomena and in the analysis of various human activities, including the study of geysers, traffic patterns, and daily water consumption [20].

2.3. The Inverse Function Method as a Way to Generate RA Samples

The task of modeling vectors of sampled RA values with circular distributions using the inverse function method can be generally formulated as follows [16]. A probability density function $f(\theta)$ of a continuous RA ϑ with domain $\theta \in [0, 2\pi)$ is given. It is necessary to generate a sample of size $1 \times N$ of this RA, that is, a sample $\Theta = \{\theta_i, i = \overline{1, N}, \ \theta_i \in [0, 2\pi)\}$.

The essence of the inverse function method for generating RA samples is that a sample of RA ϑ with a probability distribution function $F(\theta)$, defined on the interval $\theta \in [0, 2\pi)$, is obtained through a functional transformation of a random variable with a uniform distribution over the interval $a \in [0,1]$ of the form:

$$\vartheta = F^{-1}(a), \tag{2.5}$$

where $F^{-1}(a)$ – is the inverse function of $F(\theta)$ function (it is assumed that $F^{-1}(a)$ exists).

Generating a sample Θ using the inverse function method involves performing the following sequence of steps:

1. Specify the probability density function of the RA ϑ, set the parameters of the distribution law, define the sample Θ size N, and determine the number of $n << N$ approximation points (corresponding to the number of sensors in the monitoring system).
2. Determine the cumulative distribution function (CDF) of the RA as two sets of discrete, corresponding angle values. $\left(\theta_j, \ j = \overline{1, n}, \ \theta_j \in [0, 2\pi)\right)$ and the corresponding values of the cumulative distribution function (CDF) $\left(F(\theta), \ \theta = \left(\theta_j, \ j = \overline{1, n}, \theta_j \in [0, 2\pi)\right)\right)$.
3. Form the vector $\mathrm{A} = \left(a_i, i = \overline{1,N}, a_i \in [0,1]\right)$ of pseudorandom numbers with a uniform distribution over the interval [0, 1].
4. Obtain the set of values of the inverse function $\left(F^{-1}(a), a = \left(a_i, i = \overline{1, N}\right), a_i \in [0,1]\right)$. This step is advisable to perform using approximate numerical methods.
5. Obtain a vector of size $1 \times N$ of sampled RA values with distribution $F(\theta)$.
6. Construct the histogram and verify the agreement between the theoretical and experimental distributions.

2.4. Estimation of RA Probability Density Using the Method of Sample Trigonometric Moments

In mathematical statistics, various methods are used to approximate the distributions of random variables based on experimental data. Well-known approaches include approximating RV distributions using Pearson curves [21] and Johnson curves [22]. In [16], the advantages of the trigonometric moments method for approximating unimodal RA distributions were highlighted. In this paper, the effectiveness of the sample trigonometric moments method for approximating multimodal RA distributions is examined.

The trigonometric moments method is based on an important property of the characteristic function of a RA: it uniquely determines the probability density function of the RA [11–13].

$$p(\theta) = \frac{1}{2\pi} \sum_{q=-\infty}^{\infty} f_q(0) e^{-iq\theta}, \quad \theta \in [0, \ 2\pi), \tag{2.6}$$

where $i = \sqrt{-1}$, $f_q(0)$ – the trigonometric moment of order $q$of a RA relative to the zero direction (i.e., the central trigonometric moments of the RA)

Equation (2.6) represents the expansion of the function $p(\theta)$ into a Fourier series, whose coefficients are the trigonometric moments of order q.

The characteristic function of a RA ϑ, is generally defined as a complex-valued sequence of its trigonometric moments of order $q = 0, \ \pm 1, \ \pm 2, \ldots$ relative to the zero direction.

$$f_q = \mathbf{M}\{exp(in\vartheta)\} = \int_0^{2\pi} e^{iq\theta} dF(\theta),\ q = 0,\ \pm 1,\ \pm 2, ...\ , \tag{2.7}$$

where $\mathbf{M}$ – mathematical expectation operator.

The trigonometric moment f_q is expressed in Cartesian and polar coordinate systems as:

$$f_q = a_q + ib_q = \rho_q\, exp(i\mu_q), \tag{2.8}$$

where $\{a_q,\ b_q\}$ is a sequence of real numbers (the sequence of cosine and sine moments) is defined as:

$$a_q = M\{\cos(q\vartheta)\} = \int_0^{2\pi} \cos(q\theta) dF(\theta), \qquad b_q = M\{\sin(q\vartheta)\} = \int_0^{2\pi} \sin(q\theta) dF(\theta), \tag{2.9}$$

and the real numbers $\{a_q,\ b_q\}$ and $\{\rho_q,\ \mu_q\}$ are related to each other by the following relations:

$$\rho_q = \sqrt{a_q^2 + b_q^2}, \quad \mu_q = Arg(f_q), \tag{2.10}$$

As a result of performing an angular measurement experiment with RAs ϑ having a probability density function $p(\theta)$, a sample of size N is obtained: $(\theta_1,...\theta_i,...\theta_N),\ \theta_i \in [0,\ 2\pi)$. For this sample, the sample trigonometric moments of order q relative to an arbitrary direction $\alpha \in [0,\ 2\pi)$ are available for calculation.

$$\hat{f}_q(\alpha) = \frac{1}{N}\sum_{i=1}^{N} e^{iq(\theta_i - \alpha)} = \hat{a}_q(\alpha) + i\hat{b}_q(\alpha) = \hat{r}_q(\alpha) e^{i\hat{m}_q(\alpha)}, \tag{2.11}$$

where

$$\hat{a}_q(\alpha) = \frac{1}{N}\sum_{i=1}^{N}\cos[q(\theta_i - \alpha)],\ \hat{b}_q(\alpha) = \frac{1}{N}\sum_{i=1}^{N}\sin[q(\theta_i - \alpha)]. \tag{2.12}$$

The sample trigonometric function is defined as a complex-valued sequence of sample trigonometric moments $\left(\hat{f}_q(0),\ q = 0, \pm 1, \pm 2, ...\right)$.

By replacing in equation (2.6) the functions $f_q(0)$ with the corresponding sample central trigonometric moments $\hat{f}_q(0)$, and limiting the series to terms with indices $-Q_1,\ Q_1$ we obtain an expression suitable for determining an estimate of the empirical probability density function of the RA,

$$\hat{p}(\theta) = \frac{1}{2\pi}\sum_{q=-Q_1}^{Q_1} \hat{f}_q(0) e^{-iq\theta} = \frac{1}{2\pi}\left[1 + 2\sum_{q=1}^{Q_1}\left(\hat{a}_q \cos q\theta + \hat{a}_q \sin q\theta\right)\right],\ \theta \in [0,\ 2\pi). \tag{2.13}$$

Equation (2.13) defines the algorithm for calculating the empirical probability density function of the RA based on the results of their observations or on the angular histogram obtained in a simulation experiment.

2.5. Assessment of the Adequacy of Theoretical and Empirical Probability Distributions of RA

At the final stage of the experiment for generating a sample of pseudorandom angles, the adequacy of the empirical probability distribution of the random angle $\hat{p}(\theta)$, is evaluated. This is carried out by determining the absolute error, which is calculated as the difference between the specified probability density function (2.3) and the empirical one (2.13) (i.e., approximated from experimental data or from the histogram obtained during the computer experiment).

$$\delta p(\theta) = \hat{p}(\theta) - p(\theta),\ \ \theta \in [0,\ 2\pi). \tag{2.15}$$

For the function $\delta p(\theta)$ the following statistics are available: the maximum and minimum values, the mean value, the variance, and the standard deviation.

3. Implementation of the Computer Experiment for RA Generation and Analysis of the Obtained Results

3.1. General Scheme of the Computer Experiment

The procedure for conducting the experiment includes the following set of interrelated stages [8]:

1. Formulation of the experimental task and initial data, including the determination of the form of the probability distribution of the RA, its parameters, the sample size, and the number of sensors.
2. Generation of an RA sample using the inverse function method according to Section 2.2.
3. Estimation of the empirical distribution based on the obtained RA sample.
4. Assessment of the adequacy of the theoretical and empirical RA distributions.
5. Visualization and analysis of the obtained results.

3.2. Formulation of the Random Angle Sample Generation Problem and Its Implementation

According to the general methodology for conducting a computer experiment for RA generation discussed in Section 3.1, a software implementation of a pseudorandom angle generator with bimodal distributions was developed. The generator was tested by generating an RA sample with a specified distribution and comparing it with the empirical distribution obtained from the RA sample. The research task was formulated as follows.

It is necessary, using the inverse function method, to develop a software RA generator with a bimodal distribution in the MATLAB environment [18] and to study the generation process by obtaining a sample of RAs with a bimodal distribution formed by two von Mises distributions. The sample size is N=1000, and the parameters of the bimodal distribution are as follows: circular mean directions – $\mu_1 = 0{,}5$ rad and $\mu_2 = 3$ rad; concentration parameters – $k_1 = 5$ i $k_1 = 3$; and mixing parameter – $A_1 = 0{,}3$. The process of generating the RA sample should be illustrated at all stages by presenting graphs of the theoretical and empirical probability density functions of the RA, the direct and inverse cumulative distribution functions, a graphical representation of the sample, and both linear and circular histograms of the obtained RA sample.

3.3. Results of Testing the Software Implementation of the Pseudorandom Angle Generator

Below are the results of the simulation experiment for generating a RA sample based on the specified input data. Figure 3 shows the graphs of the given (theoretical) distributions: (a) the densities of the two von Mises distributions obtained according to (2.1); (b) the multimodal probability density function according to (2.4).

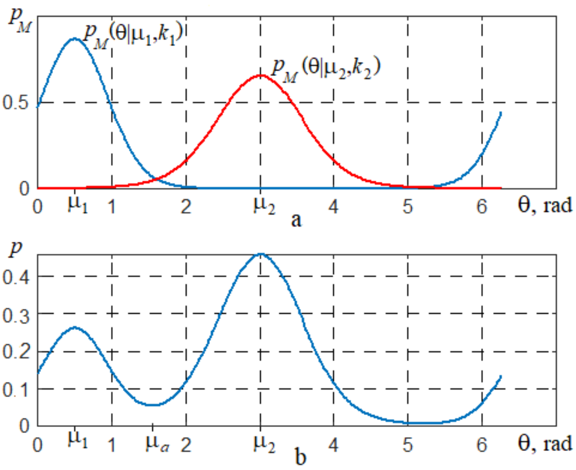


Fig. 3. Graphs of the von Mises probability densities for the RA components (a) and the multimodal circular probability density function (b)

The modes of the bimodal distribution coincide with the modes of the unimodal distributions, with the primary mode at $\mu_2 = 3$ read, the secondary mode – $\mu_1 = 0{,}5$ rad, and the antimode at $\mu_a \approx 1.54$ rad.

Figure 4a shows the cumulative distribution function (CDF) of the von Mises distribution $F(\theta)$, and Figure 4b shows its inverse function $F^{-1}(a)$. The function $F(\theta)$ was obtained by integrating the probability density function $p(\theta)$, using the trapezoidal method.

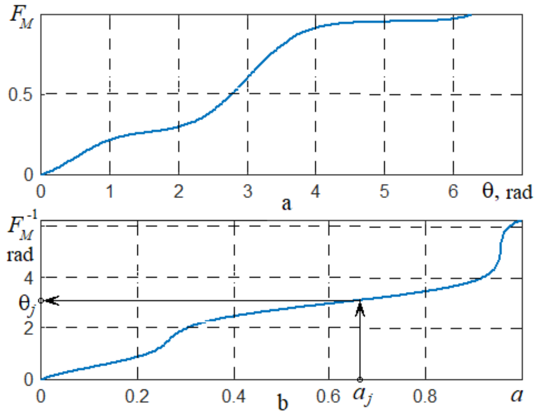


Fig. 4. Graphs of the cumulative multimodal distribution $F(\theta)$ (a) and its inverse distribution $F^{-1}(a)$ (b)

Figure 4b illustrates the idea of generating a RA sample using the inverse function method, showing the determination of a single sample element $\theta_j \in [0,\ 2\pi)$, я corresponding to a pseudorandom number $a_j \in [0,\ 1]$ with a uniform distribution.

The entire RA sample of size *N*=1000 is shown in Figure 5a, and the corresponding linear histogram is shown in Figure 5b. The histogram was constructed by uniformly dividing the entire range of angle values into 20 intervals, each with a width of $0.1\pi \approx 0.314$ rad. A distinctive feature of the software implementation of the RA generator is that the sample generation itself is carried out using the **spline** function, which is applied with a nonstandard ordering of arguments as follows:

- the vector of probability function values $F(\theta)$ of size *n*;
- the vector of angle values for which the function $F(\theta)$ of size n is evaluated;
- the vector of pseudorandom numbers with a uniform distribution of size $N >> n$.

Such an implementation of the inverse function method does not require the analytical determination of the inverse function for generating the RA sample.

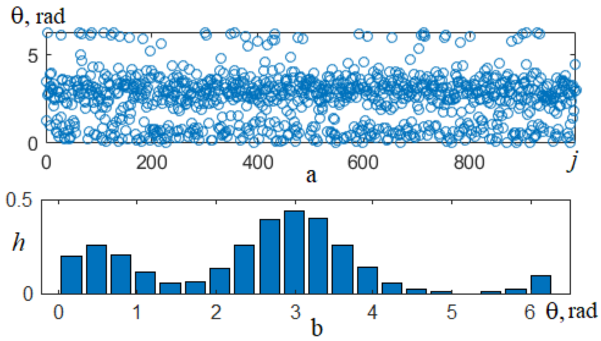


Fig. 5. Sample of random angles with a bimodal distribution (a) and the corresponding linear histogram (b)

A more natural and illustrative way to represent circular data is a circular histogram **(polar wedge diagram)** [5]. Such a diagram for the obtained sample is shown in Fig. 6a.

In this diagram, the observation results are represented by sectors with a common vertex at point *O*. The angles of the sectors correspond to the selected class intervals of size (0.1π), while the radius vectors are equal to the heights of the bars in the linear histogram. In the diagram shown in Fig. 6a, the angle values are plotted counterclockwise from the *Ox* axis.

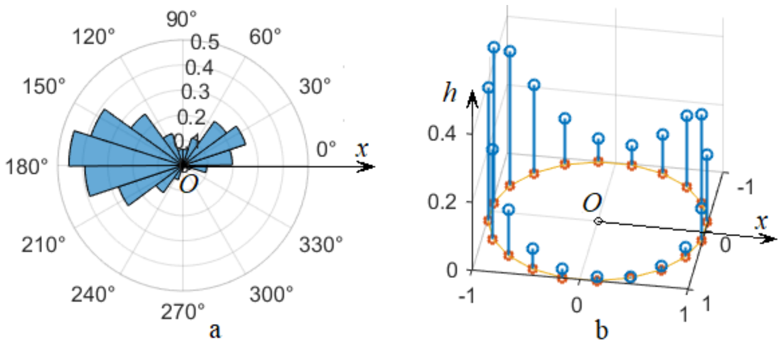


Fig 6. Circular histogram of the RA sample (a) and its representation as a 3D graph (b)

At the final stage of the experiment, the correspondence between the distribution of the generated sample and the theoretical distribution was verified. For this purpose, the empirical probability density of the bimodal RA was first determined from the obtained angle sample according to equation (2.13). The resulting graph of this density is shown in Fig. 7a.

The graph of the absolute error, determined according to equation (2.15) as the difference between the specified (theoretical, a priori) and the empirical (a posteriori) probability density functions of the RA, is shown in Fig. 7b. This graph demonstrates that the absolute value of the error does not exceed 0.005 over the entire interval of angles $[0,\ 2\pi)$, which indicates a good agreement between the theoretical and experimental bimodal RA distributions.

In addition, the adequacy of the theoretical and experimental distributions was evaluated using the Kolmogorov criterion according to the methodology described in [8]. The tested hypotheses are formulated in terms of probability distribution functions. The null and alternative hypotheses regarding the distributions are defined as follows:

$$H_0:\ F(\theta) = F_e(\theta),$$
$$H_a:\ F(\theta) \neq F_e(\theta),$$

where $F_e(\theta)$ – the empirical distribution obtained from the generated RA sample.

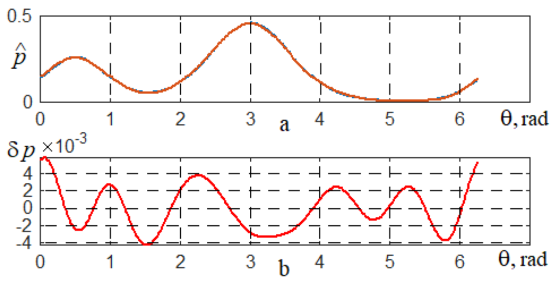


Fig. 7. Graph of the empirical probability density function $\hat{p}(\theta)$ and its absolute error δp

For the performed simulation experiment with the bimodal distribution, the following results were obtained:

- the maximum value of the absolute difference between the experimental $F_e(\theta)$ and theoretical $F(\theta)$ probability density functions of the RA was $D = 0.00528$;
- the critical value of the probability parameter λ in the Kolmogorov test was – $\lambda = D\sqrt{N} = 0.00528 \cdot \sqrt{1000} \approx 0.167$;
- for the chosen significance level $\varepsilon = 0.05$ the critical value of the parameter $\lambda_o = 1.358$ was determined;

Overall, it was concluded that the distributions are adequate, since the inequality $\lambda \approx 0.167 < \lambda_o = 1.358$ is satisfied.

3.4. Discussion of the Obtained Results

Modern statistical methods for processing circular data have significant potential for use in the energy sector, particularly in environmental monitoring systems for energy facilities to determine the spatial distribution of harmful substances in the near-surface layers of the atmosphere around these objects. Such systems require the verification of their algorithmic and software components, which in turn involves the development of RA generators capable of producing test sequences with complex probability distributions.

The methodology for generating RA samples proposed in this paper is based on a combination of the inverse function method and the trigonometric moments method, known from the section of mathematical statistics devoted to the analysis of circular data. The software generator developed on this basis is capable of producing high-quality samples over a wide range of RA sample sizes. Its applicability for generating samples with multimodal angular distributions has been experimentally demonstrated. The effectiveness of the generator was verified using the task of generating an RA sample with a bimodal distribution, defined as a weighted sum of two independent RAs with von Mises distributions. In the conducted experiment, the absolute error, calculated as the difference between the theoretical and empirical probability density functions of the RA, did not exceed:

for a sample of size N=1000 the value was $|\delta p| < 0.005$, and for a sample of size N=100 it was $|\delta p| < 0.03$.

In the example presented, the trigonometric moments method plays a supplementary role and is used only to assess the adequacy of the RA generator. However, this method can also have independent value in tasks involving the evaluation of angular measurement results, particularly as a method for constructing interval estimates of angular observations when the experimental data size is limited, making it impossible to perform formal hypothesis testing on the distributions of circular data.

4. Conclusions

The task of verifying the algorithmic and software components of systems for processing circular data requires the development of RA generators capable of producing test samples with complex probability distributions. Such a need particularly arises in atmospheric monitoring systems around energy facilities, which allow determining the spatial distributions of harmful substances from pollution sources. This paper examines the methodological basis for generating RA samples with multimodal circular distributions, which is based on a combination of the inverse function method for RA sample generation and the sample trigonometric moments method. This approach allows obtaining empirical circular distributions from sample data and verifying the adequacy of the generated data.

The Kolmogorov criterion was adapted to evaluate the quality of the RA generator by testing the adequacy of the specified theoretical distribution against the empirical probability density distribution obtained from the generated data.

A software implementation of an RA generator with a multimodal distribution has been developed. The generator was experimentally tested by generating samples with a bimodal distribution formed from two RAs with von Mises distributions. The developed software implementation utilizes features of the MATLAB environment, which enabled the realization of the inverse function method without requiring its analytical determination.

The developed RA software generator can be valuable both for conducting computer experiments with random angular data and for integration into software designed to process circularly distributed data.

Further research will focus on investigating methods for estimating sample statistics of bimodal distributions.

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ГЕНЕРАТОР ВИПАДКОВИХ КУТІВ У СИСТЕМІ МОНІТОРИНГУ АТМОСФЕРИ НАВКОЛО ОБ'ЄКТІВ ЕНЕРГЕТИКИ

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Анотація. *В умовах воєнного стану ускладнюється підтримання балансу потужності в енергосистемі, штатних режимів роботи обладнання об'єктів енергетики, дотримання* *відповідної якості використовуваних викопних енергоресурсів тощо, що призводить до підвищення викидів шкідливих речовин в атмосферу. Тому питання розвитку та удосконалення засобів моніторингу довкілля об'єктів енергетики набуває особливої актуальності. В системах екологічного моніторингу первинна вимірювальна інформація формується системою сенсорів, розміщених навколо джерел забруднюючих речовин. Одним з важливих завдань аналізу експериментальних даних є оцінювання розподілу концентрації забруднюючих речовин в околі об'єктів енергетики у полярних координатах. Для його вирішення до складу алгоритмічно-програмного забезпечення систем моніторингу включають модулі статистичного аналізу кругових даних. Завдання верифікації таких модулів потребує створення генераторів випадкових кутів, які були б спроможні формувати тестові вибірки випадкових кутів зі складними розподілами ймовірностей. У статті розглянуто методологічний базис розв'язання завдань комп'ютерних експериментів з випадковими кутами, обґрунтовано використання полімодальних розподілів ймовірностей в задачах аналізу кругових даних моніторингу атмосфери, запропоновано генератор випадкових кутів та виконано його верифікацію на прикладі генерування вибірки з бімодальним розподілом. Для оцінювання адекватності теоретичного та емпіричного розподілів ймовірностей випадкового кута використано критерій Колмогорова. Розроблений генератор дає змогу отримувати вибірки випадкових кутів з розподілами, які задані аналітично, таблично чи графічно (гістограмою), та може бути використаний для тестування алоритмічно-програмного забезпечення в системах статистичного аналізу кругових даних.*

Ключові слова: генератор випадкових кутів, комп'ютерний експеримент, метод тригонометричних моментів, бімодальні розподіли випадкових кутів.

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