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METHOD FOR DETERMINING AVERAGE VOLUME PARAMETERS OF THE DYNAMICS OF THE ELECTROTHERMAL STATE OF MEDIA USING THE GAUSS – OSTROGRADSKY THEOREM

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Abstract. *In this paper, using the methodology of spatial averaging of partial differential equations, based on the application of the divergence theorem, the average volume parameters of the dynamics of the electrothermal state of media associated with the geometric shape of the medium limited in space are determined. It is shown that the volume-averaged parameters of the dynamics of the electrothermal state of media can be represented by the parameters averaged over the surface surrounding the volume. This allows us to significantly simplify the analysis of the final results of research into the processes of interaction of solar radiation with the active surfaces of elements of solar energy systems, in particular, the influence of the surrounding medium.*

Keywords: *electrothermal state of the medium, heat and mass transfer, vector analysis, partial differential equations, divergence theorem.*

МЕТОД ВИЗНАЧЕННЯ СЕРЕДНЬООБ'ЄМНИХ ПАРАМЕТРІВ ДИНАМІКИ ЕЛЕКТРОТЕПЛОВОГО СТАНУ СЕРЕДОВИЩ З ВИКОРИСТАННЯМ ТЕОРЕМИ ГАУСА – ОСТРОГРАДСЬКОГО

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Анотація. *Анотація. В роботі з використанням методології просторового усереднення рівнянь в частинних похідних на основі застосування теореми про дивергенцію визначено середньооб'ємні параметри динаміки електротеплового стану середовищ, пов'язаних з геометричною формою обмеженого в просторі середовища. Показано, що усереднені за об'ємом параметри динаміки електротеплового стану середовищ можливо представити через усереднені за поверхнею, що оточує об'єм, параметри. Це дозволяє значно спростити аналіз кінцевих результатів досліджень процесів взаємодії сонячного випромінювання з активними поверхнями елементів сонячних енергетичних систем, зокрема, впливу оточуючого середовища.*

Ключові слова: *електротепловий стан середовища, тепломасоперенос, векторний аналіз, рівняння в частинних похідних, теорема про дивергенцію.*

Introduction. In the 60s of the 20th century, at the Institute of Electrodynamics of the National Academy of Sciences of Ukraine, on the initiative of Doctor of Technical Sciences, Professor Yury Petrovych Yemets, research was proposed into the methods for calculating and analyzing the averaged electrophysical characteristics of media with different spatial structures of inhomogeneities and taking into account complicating factors, in particular, anisotropy caused by the Hall effect. The results of research, carried out over the next 50 years by his students and colleagues, made it possible to obtain original data on the anomalous nature of the effect of electrical conductivity anisotropy on the averaged electrical and electrophysical characteristics of heterogeneous low-temperature plasma, semiconductors and other media used in various electrical engineering and power generation devices. What is important in these works is that they were based on non-trivial methods of mathematical physics, and the dependencies for the averaged characteristics were compared with data on the local distribution of fields in the region of inhomogeneities and their surrounding media.

It is known that during the interaction of solar radiation with any media, one of the basic processes is the conversion of solar radiation energy into thermal energy, which leads to an increase in their temperature. For solar thermal energy systems, such a process is positive and necessary from a practical point of view. For photovoltaic systems designed to produce electrical energy, heating of active elements has a negative effect, since for all known photoconverters, an increase in their temperature leads to a decrease in the coefficient of conversion of solar radiation energy.

Today, models and methods for calculating the processes of interaction of solar radiation with active surfaces of elements of solar thermal and photovoltaic systems have been developed in sufficient detail at the level of Maxwell's equations and the partial differential equation of thermal conductivity, which describe the distribution of the corresponding parameters (solar radiation intensity, heat release density, temperature and temperature gradients) in space and time. In particular, using the synergetic methodology of I. Prigozhin [1, 2], the main causes of the emergence of self-oscillating modes as a result of the emergence of instabilities in nonlinearly related processes of energy conversion of renewable sources have been formulated [3], an analysis of the stability of nonlinear thermal processes in media with different spatial structure of disturbances was performed [4], which, under conditions of instability, can lead to the formation of spatially inhomogeneous structures with the possible realization of fluctuations in time and a corresponding decrease in the reliability of functioning and resource due to the processes of degradation of the structure of functional materials. It should be noted that existing experimental methods of electron microscopy and X-ray phase analysis allow determining the morphology of only the surface, and do not provide the opportunity to determine the volumetric distributions of the above parameters.

Since in solar thermal and photovoltaic energy, mainly integral parameters are experimentally determined, namely: the intensity of solar radiation, the average temperature of the active surfaces of solar collectors and photoconverters, and in some cases the initial temperature of the coolants when selecting heat in solar collectors and photoconverters, there is a practical need to develop a method for transitioning from models described by partial differential equations to models described by ordinary differential equations for volume-averaged parameters. With this approach, the question of how to determine the volume-averaged stress fluxes associated with the geometric shape of a spatially confined medium and the influence on this medium through the corresponding boundary conditions is open.

To solve this issue, it is proposed to use the methodology of transition from a local description of the change of parameters in time and space to a description of the change of volume-averaged parameters only in time. This methodology, the essence of which lies in the theory of spatial averaging of partial differential equations in space, was developed in the works of academicians of the National Academy of Sciences of Ukraine V.O. Marchenko and E.Ya. Khruslov, authors of the theory of averaging boundary value problems in mathematical physics [5]. The basis of this methodology is the use of integral theorems of vector analysis, in particular the fundamental theorem of divergence (Gauss-Ostrogradsky theorem) [6].

Setting the problem. The basic initial and simplest model of the electrothermal state of the medium in the form of a parabolic equation is considered

$$\frac{\partial w}{\partial t} + \nabla \cdot \vec{q} = q_v, \qquad (1)$$

where: $w = \rho c_p T$ – specific heat, a ρ, c_p, T – density, specific heat capacity and temperature, respectively;

$\vec{q}$ – heat flow density;

q_v – volume density of heat generation due to absorption of electromagnetic radiation.

The magnitude of the heat flow vector $\vec{q}$ is determined by the Fourier law [6]

$$\vec{q} = -\lambda \nabla T, \qquad (2)$$

where λ – coefficient of thermal conductivity.

The volume density of heat release q_v is determined by the relation

$$q_v = -\nabla \cdot \vec{\Pi}, \qquad (3)$$

where $\vec{\Pi}$ – Umov-Poynting vector ($\vec{\Pi}=\vec{E}\times\vec{H}$), a $\vec{E},\vec{H}$ – intensity of the electric and magnetic fields in the medium, respectively.

The methodology of reducing the volume heat release density q_v to a divergent form. Because q_v is a scalar quantity with dimension W/m^3, then it is naturally to analyze the function $\nabla\cdot\vec{\Pi}$, which is also scalar with dimension W/m^3. For this purpose, according to [5], we use the operation

$$\nabla\cdot\vec{n}=\nabla\cdot\left(\vec{E}\times\vec{H}\right)=\vec{H}\cdot\left(\nabla\times\vec{E}\right)-\vec{E}\cdot\left(\nabla\times\vec{H}\right). \quad (4)$$

Using the system of Maxwell's equations, it is possible to show that the relation holds

$$\nabla\cdot\vec{n}=-\vec{H}\cdot\left(\frac{\partial\vec{B}}{\partial t}\right)-\vec{E}\cdot\left(\sigma\vec{E}+\frac{\partial\vec{D}}{\partial t}\right), \quad (5)$$

where: $\vec{B}=\mu\vec{H},\ \vec{D}=\varepsilon\vec{E}$ – induction of magnetic and electric fields, respectively;
σ,μ,ε – electrical conductivity, magnetic and dielectric permeability.

At $\mu,\varepsilon - const$ from (5) the relation follows

$$\nabla\cdot\vec{\Pi}=-\sigma\vec{E}^2-\frac{\partial\left(w_\mu\right)}{\partial t}-\frac{\partial\left(w_\varepsilon\right)}{\partial t},\ w_\mu=\frac{\mu\vec{H}^2}{2},\ w_\varepsilon=\frac{\varepsilon\vec{E}^2}{2}. \quad (6)$$

In terms of physical meaning and dimensionality, the functions $w_\mu,\ w_\varepsilon$ correspond, in sinusoidal current circuits, to the energies stored in inductances and capacitancesand do not contribute to thermal energy.

Therefore, volumetric heat release value $q_v=\sigma\vec{E}^2$, i.e. $q_v=-\nabla\cdot\vec{\Pi}$.

From a mathematical point of view, this is explained by the fact that functions $\frac{\partial w_\mu}{\partial t},\frac{\partial w_\varepsilon}{\partial t}$ are sinusoidal, which when averaged over time T for a period, that significantly exceeds the period of electromagnetic oscillations, are equal to zero.

Thus, the use of the divergence theorem [5] taking into account the fact that $q_v=-\nabla\cdot\vec{\Pi}$, given $\rho,c_p - const$, leads to the following expression for the average temperature $T_{cp}=\int_V TdV/V$

$$\frac{dT_{cp}}{dt}=\frac{\left(Q_{vcp}-Q_{scp}\right)S}{\rho c_p V},$$

$$Q_{vcp}=-\frac{\oiint_S \vec{\Pi}\cdot d\vec{S}}{S},\ Q_{scp}=\frac{\oiint_S \vec{q}\cdot dS}{S}. \quad (7)$$

Here V – volume bounded by a closed surface S;

$d\vec{S}=\vec{\Pi}\left|d\vec{S}\right|$ – vector of the elementary plane to the surface S with unit normal $\vec{\Pi}$;

Q_{vcp} – average over surface S amount of thermal energy of the electromagnetic field falling on the surface S;

Q_{scp} – average over surface S amount of thermal energy that is transferred to the external environment.

Equation (7) allows us to determine the growth dynamics of the average temperature over time (at $Q_{vcp}>Q_{scp}$), temperature value T_{cp} in saturation mode (at $\frac{dT_{cp}}{dt}=0$) and its reduction (at $Q_{vcp}<Q_{scp}$) after the electromagnetic field is removed due to the dissipation of thermal energy into the surrounding environment.

It is worth noting that the nature of change T_{cp} over time depends essentially on the relationship between V i S, which demonstrates different patterns when the characteristic dimensions of the environment and its shape change.

It was established that dependence (7) is quantitatively or qualitatively confirmed by experimental data for a limited range of power values Q_{vcp}, Q_{scp}, geometric parameters V, S and physical characteristics ρ,c_p.

Note that this provides a basis for applying the Marchenko-Khruslov methodology to other media, power values, and equipment sizes. A crucial aspect here is that the parameters Q_{vcp}, Q_{scp} are significantly easier to determine experimentally or calculate on the surface S rather than determining their local distribution within the volume V. This conclusion, in particular, is confirmed in our work on the dynamics of photovoltaic cell heating under solar radiation [8 - 10], heating of various composite materials under microwave radiation [11] and air heating in localized areas using locally distributed electric heaters, as well as in electromass transfer processes [12].

Conclusions. It has been established that it is advisable to use the Gauss-Ostrogradsky theorem to average the parameters of the dynamics of the electrothermal state over the volume of the media. For this, a necessary condition is the possibility of representing differential operators in spatial coordinates in the original models in a divergent form. It has been demonstrated that the volume-averaged parameters of the dynamics of the electrothermal state of the media can be expressed through parameters averaged over the surface surrounding the volume. This greatly simplifies the analysis of final results and practical conclusions, in particular, the influence of the surrounding medium.

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